15. Estimation of Satellite Orbits and Earth Orientation Parameters

15.1   Introduction

Today most users have no need to compute precise GNSS orbits since highest precision orbits are available from the IGS (with different latencies). It is, in fact, close to impossible to come up with better orbits than those produced by the IGS. See Sections 2.2.1 and 4.12 for more information on IGS orbits and server locations where the products are available. Nevertheless orbit improvement is one of the basic functions of the *Bernese GPS Software*. At CODE as well as at a few other IGS ACs the *Bernese GPS Software* is used to compute precise GNSS orbits for every day following a daily schedule.

Orbit improvement is possible for GPS and GLONASS satellites (and in future also for GALILEO satellites), but also for low Earth orbiters equipped with GNSS tracking receivers. The *Bernese GPS Software* allows orbit determination in principle also with SLR observations or with combined SLR and GNSS observations (combining the technique specific normal equations with ADDNEQ2). This chapter provides information on the sequence of steps needed for orbit improvement focusing on orbit improvement for GNSS satellites based on microwave tracking data. The procedure is, however, more general and can be applied equally well for precise orbit determination for LEOs or based on SLR observations. Important differences are outlined in the text.

Data from a globally distributed network should be used for a reliable estimation of GNSS orbit parameters. Only a subset of orbit parameters may be improved with a regional network. Usually, double differenced observations are processed for orbit determination with the *Bernese GPS Software* since ambiguity resolution positively impacts the quality of the estimated orbits. A relatively dense global network is, however, required in this case.

GNSS tracking data acquired from receivers on the Earth's crust allow to estimate a subset of Earth orientation parameters because the GNSS satellite constellations form a realization of an inertial system (or quasi-inertial system considering the Earth orbiting around the Sun). Because the satellites sense the gravity field of the Earth, their motion can as well be used to obtain information on the Earth's center of mass with respect to the crust-fixed terrestrial reference frame. Information on the estimation of Earth orientation parameters and geocenter coordinates is given in Section 15.4.

15.2   Fitting Precise Files and Predicting Orbits

Section 5.4 describes the procedure on how to prepare precise orbit information for use within the *Bernese GPS Software*. The same procedure may be employed to fit a series of precise files as well as to predict the adjusted orbit into the future (or the past). For fitting n precise files, each of them is first converted to tabular format as described in Section 5.4.1.

In order to get the tabular orbits in consistent representations of the inertial frame, special attention has to be paid to the Earth rotation information used because UT1-UTC information from successive ERP files may not be continuous (see Section 15.2.1). Depending on the application you may use ERP information from IERS, e.g., Bulletin A or C04 covering the time period for which you want to get the orbit fit and prediction (see Section 5.2.2).

To fit the n tabular files, select them all in the field "Start with tabular orbits" in program ORBGEN. For predicting the orbit into the future (or into the past) simply shift the right (resp. left) boundary of the time window accordingly. The complete radiation pressure model (nine parameters) is well suited to obtain a good fit over several days. You get the standard orbit file adjusting the selected tabular files and covering the specified time window as output. You may also generate a so-called LST file (see Figure 22.59) that contains a table of the fit rms values for each satellite and each input orbit file. This file may be used to compute accuracy information for a precise file header (see Section 15.2.2).

Note that instead of tabular files precise files may directly be used as input for ORBGEN. For each input precise file a corresponding ERP file has to be available with the same name as the precise file (but different extension).

15.2.1   Preparation and Extrapolation of ERP Information

The UT1-UTC corrections provided with the Earth orientation information generated based on GNSS tracking data are referenced to an external value independently for each pole file by fixing, e.g., the first value to Bulletin A. The reason is that GNSS is insensitive to this correction (see Section 15.4). As a consequence precise orbit files converted to the inertial frame exhibit jumps at pole file boundaries that may reach more than 10 cm.

Two programs available in the *Bernese GPS Software* can be used to align the UT1-UTC values from different ERP files to a common reference: POLXTR and POLINT. The first of the two programs, available at "Menu>Orbits/EOP>Handle EOP files>Concatenate IERS pole files", allows for a sophisticated selection of the records from different input IEP files (ERP files in IERS format, see Section 4.5) to be concatenated to a single Bernese pole file. To use the output pole file with ORBGEN select the corresponding option in the field "Handling of UT1-UTC". The program may also be used to extrapolate pole information into the future using a polynomial fit. We recommend to use a linear extrapolation only. Observe that UT1R-UTC is fitted and extrapolated (i.e., the Earth rotation angle reduced by tidal variations) and not UT1-UTC. For additional information on the use of program POLXTR and the appropriate setting of the options we refer to the on-line help.

Program POLINT is only available using the RUNGPS command (see Section 18.8). It allows to concatenate input pole files (in IERS format) in a straightforward way. Activate option "Continuity between sets" in order to get a continuous time series in UT1-UTC. Do not activate option "Restitute subdaily pole model" which is available for tests only.

15.2.2 Conversion of Standard Orbit Files to Precise Files

A standard orbit file can be converted to precise file format using the program STDPRE ("Menu >Orbits/EOP>Convert standard to precise orbits"). The program performs the transformation from the inertial to the Earth-fixed reference frame. For this transformation the same nutation and ERP information must be used as for the generation of the standard orbit file in the previous steps. You may include satellite clock corrections in the precise file by specifying a Bernese clock file in the field "Satellite clocks". Missing clock values are substituted with "999999.999999".

If a satellite problem file (see Section 22.4.6) is specified in the corresponding field in panel "STD-PRE 1.1: General Files", satellites marked as bad for the specific time period are not written to the output precise file. The satellite problem file also provides the maneuver epoch for repositioned satellites. The orbit positions corresponding to the correct arc before and after the maneuver are then written to the output file.

There are two possibilities to include accuracy information in the precise file header. Accuracy codes reflecting the formal orbit accuracy as estimated by GPSEST or ADDNEQ2 may be derived from an input orbital element file (see Section 15.3.1.2). Program STDPRE reads the formal error du of the argument of latitude (in radians) for each satellite and converts it to an accuracy code using the formula $acc = NINT(\log(a \cdot du)/\log(2))$ where $a = 2.65 \cdot 10^{10}$ m is the semimajor axis.

Alternatively program PREWEI ("Menu>Orbits/EOP>Set accuracy codes in precise orbits") may be used to include accuracy information derived from the fit rms values gathered from a LST-file written by ORBGEN. Such a file is generated when specifying a filename in the field "Summary file" in panel "ORBGEN 2: Result and Output Files" and fitting precise files covering several days with ORBGEN. The accuracy information then represents the 'modelability' of the orbits that is in general more realistic than the information obtained from the formal errors in the orbital elements file.

15.3   Orbit Improvement

Orbit improvement (sometimes also called orbit determination) is the process of improving orbital parameters using observations. In the *Bernese GPS Software* two programs are involved in orbit improvement: ORBGEN and GPSEST. In addition program ADDNEQ2 may be used to combine orbit information from several arcs. Orbital parameters that may be improved include the six Keplerian osculating elements (initial conditions at the beginning of the arc), up to nine parameters of the CODE empirical radiation pressure model (see Section 2.2.2.3), and stochastic pulses (velocity changes) at predefined epochs (see Section 2.2.2.4).

Within the *Bernese GPS Software* estimated orbits always refer to the satellite's center of mass. Information on the satellite antenna phase center offset is obtained from the satellite information file in conjunction with nominal satellite attitude (or, for LEOs, alternatively from an attitude file, see Section 16.2.3).

15.3.1   The Procedure for Orbit Improvement in the *Bernese GPS Software*

Orbit improvement in the *Bernese GPS Software* involves three steps:

(1) **Preparation of a priori orbit information:**

A priori orbit as well as the partial derivatives of the orbit positions with respect to the parameters to be estimated are generated with program ORBGEN by numerical integration of the equations of motion and of the variational equations. The information is written to a standard orbit (default extension STD) and a so-called radiation pressure file (default extension RPR).

(2) **Estimation of improvements for orbit parameters:**

Program GPSEST reads the a priori orbit and the derivatives from these two files, builds up the normal equations in a loop over all observations, and solves the equation system to obtain the parameter improvements. The improved orbital parameters are written to a so-called element file (default extension ELE) refering to the start epoch of the a priori orbit (osculation epoch).

(3) **Update of orbit:**

ORBGEN reads the element file written by GPSEST and integrates the orbit numerically based on the improved orbit parameters. The output is a standard orbit for a specified time interval.

The last two steps may be iterated if the a priori orbit deviates too much from the 'true' orbit. Subsequent steps may include a combination of a sequence of orbital arcs generated by GPSEST to longer arcs using program ADDNEQ2 and the conversion of the improved standard orbit into a precise file. The separate steps are described in more details in the following sections.

15.3.1.1 Prepare A Priori Orbit Information

Let us first state that if you improve orbits it is in principle not important whether you start from broadcast or precise orbit information to create an a priori standard orbit. If your a priori orbit is, however, only good to about 20 m you should definitely go through a second iteration step, i.e., repeat the orbit improvement steps described in Section 15.3.1.2 and 15.3.1.3 with the orbit generated in the first iteration step, which is now certainly within 0.1 m of the final results.

You may thus generate the a priori orbit starting from broadcast information as described in Section 5.3 or from a precise orbit file as described in Section 15.2 (e.g., obtained by prediction from previous days). To generate the necessary files you may follow exactly the procedure described in Section 5.4.2 with the standard ORBGEN parameter settings. The output orbit information has to cover the complete time interval for which you want to perform orbit improvement with a single arc. It is recommended to restrict to daily arcs.

Not only the equations of motion but also the so-called variational equations (derivatives of satellite positions with respect to orbit parameters, see Section 2.2.2.5) have to be integrated numerically for orbit improvement. Specify a filename in the field "Radiation pressure coeff." in panel "ORBGEN 2: Result and Output Files" in order to invoke the integration of the variational equations.

The integration process writes the polynomial coefficients for each satellite, each component, and each integration subinterval into the standard orbit file specified in the field "Standard orbits" and those for all the partials into the radiation pressure file (default extension RPR). However, if the coefficients for the partial derivatives were saved in the same way as those for the satellite positions, the file length of the radiation pressure files would be 15 times the size of the standard orbit files –

which is a waste of disk space! This is true in particular if one takes into account that the accuracy requirements for the partials are by no means as stringent as those for the orbits.

The procedure used in Version 5.0 seems to be an optimum: whereas the variational equations are solved using exactly the same interval subdivision and the same polynomial degree as for the integration of the primary equations, it is possible and advisable to change the polynomial degree and the subinterval length for storing the coefficients associated with the variational equations. In practice we use for GNSS satellites a subinterval length of six hours and a polynomial degree of 12 for storing the coefficients for the partials (options "Length of interval" and "Polynomial degree" related to the section "VARIATIONAL EQUATION" in panel "ORBGEN 3.2: Options", see Figure 5.6). Through this procedure we have the partials available in the files with sufficient precision (6 to 8 significant digits) without wasting too much disk space. For LEOs a subinterval length of 0.2 hours and a polynomial degree of 10 seem to be appropriate.

15.3.1.2   Improve Orbit Parameters

The actual orbit improvement has to be set up in program GPSEST. We recommend to use program GPSEST with data spans of at maximum one day. If you actually want to produce longer arcs, use program ADDNEQ2 to combine the one-day arcs, see Section 15.3.2.
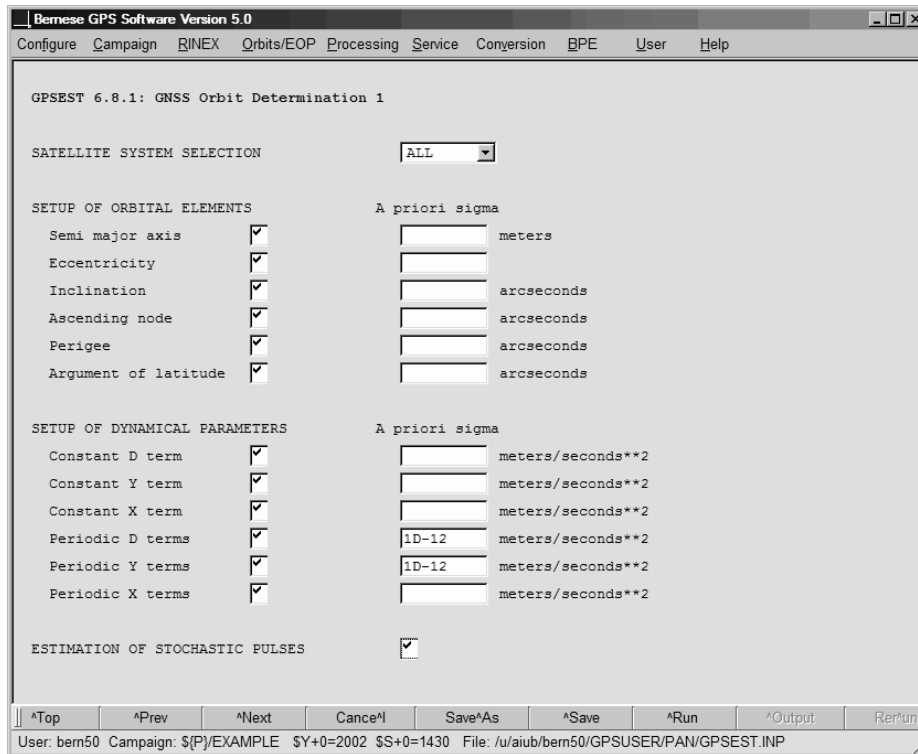
To allow for orbit parameter estimation specify the names of the a priori standard orbit file in field "GNSS standard orbits" in panel "GPSEST 1.1: Input Files 1" and of the radiation pressure file in field "GNSS orbit partials" in panel "GPSEST 1.2: Input Files 2". For LEO orbit improvement mark the checkbox "LEO data processing" in panel "GPSEST 1.1: Input Files 1" and specify the names of the two a priori files in the fields "Standard orbit(s)" and "Orbit partials" in the section "LEO INPUT FILES" of panel "GPSEST 1.2: Input Files 2". The names of the output orbital element file is specified in the fields "GNSS orbital elements" resp. "LEO orbital elements" in panel "GPSEST 2.2: Output Files 2".

Orbit parameter estimation is enabled by marking the checkbox "GNSS or LEO orbit determination" in panel "GPSEST 5.2: Setup of Parameters and Pre-Elimination 2" (the option gets active if a radiation pressure input file is specified). A subsequent panel is then activated that is specific for the setup of orbit parameters.

In the first panel "GPSEST 6.8.1: GNSS Orbit Determination 1" (see Figure 15.1) you may specify whether you want to estimate orbit parameters for GPS only, for GLONASS only, or for all satellite systems ("SATELLITE SYSTEM SELECTION"). The option has to be set to ALL for LEOs. In the same panel you can setup each individual osculating element (section "SETUP OF ORBITAL ELEMENTS") and dynamical parameter (radiation pressure parameters, section "SETUP OF DYNAMICAL PARAMETERS") and a corresponding a priori constraint.

In order to preserve all options for future runs with program ADDNEQ2 we recommend that you set up all (osculating and dynamical) orbit parameters but that you tightly constrain parameters that you want to fix for a particular experiment. You are then able to change the constraining in runs based on normal equations. In our analysis at CODE we freely estimate all parameters but constrain for GNSS satellites the periodic dynamic parameters in the directions D and Y because correlations with Earth orientation parameters would otherwise affect the estimates of length of day (see settings in Figure 15.1).

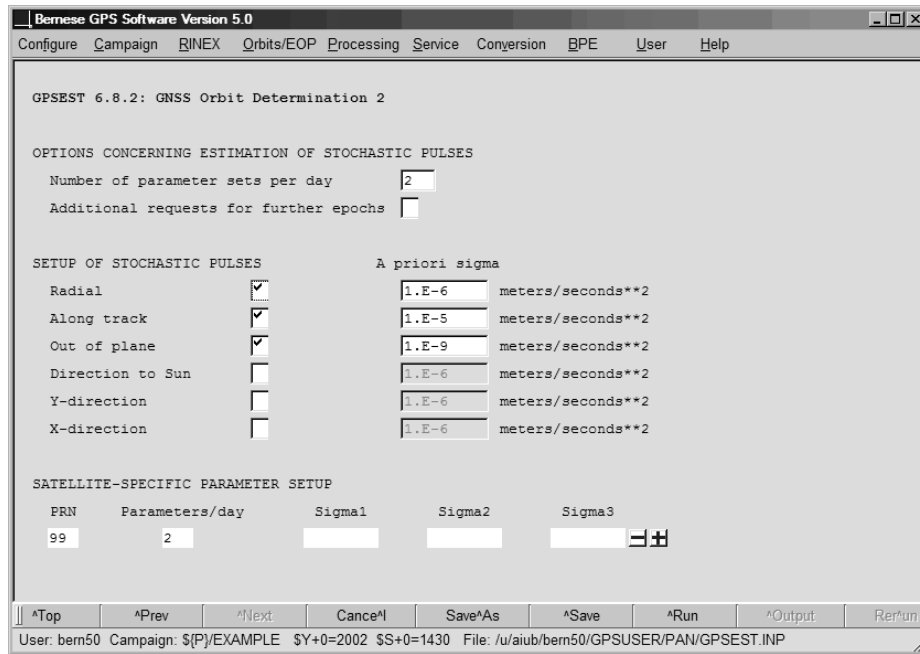The option "ESTIMATION OF STOCHASTIC PULSES" activates the panel "GPSEST 6.8.2: GNSS

Figure 15.1: Options for defining the setup of orbital parameters in GPSEST.

Orbit Determination 2" (see Figure 15.2) that allows for a detailed setup of pseudo-stochastic parameters (velocity changes at pre-defined epochs in three orthogonal directions).

The number of equally spaced epochs per satellite and per day for which stochastic parameters shall be set up may be specified in option "Number of parameter sets per day". If you enter e.g. 2, parameter sets would in principle be set up at 00 UT, 12 UT, 24 UT, etc. The parameters at the beginning and end of the arc are, however, not set up. For a 1-day arc, therefore, only the pulses at noon are estimated. Pulses at the day boundaries may later be set up in ADDNEQ2 if you want to combine 1-day arcs into n-day arcs (Section 15.3.2). For GNSS satellites we recommend to set up two sets of stochastic parameters per day. For LEOs more pseudo-stochastic parameters may be set up, e.g., every 15 min (96 parameter sets per day). Note that stochastic pulse epochs have to coincide with an integration interval boundary in ORBGEN.

Section "SETUP OF STOCHASTIC PULSES" in the panel allows to specify the direction for which pseudo-stochastic pulses shall be set up as well as their default a priori constraint (with respect to zero). In the example displayed in Figure 15.2 pulses are set up in radial, along-track, and in out-of-plane directions. To cope with correlations with Earth rotation parameters the out-of-plane direction is actually constrained to (almost) zero.

The section "SATELLITE-SPECIFIC PARAMETER SETUP" in the same panel can be used to specifically define satellite-specific requests concerning the setup of pseudo-stochastic pulses. You may include lines with individual satellite PRN number, "99" for all satellites, or "98" for eclipsing satellites. If "97" is specified, stochastic pulses are set up for eclipsing satellites only for the epochs about 45 minutes after the satellites leaving the Earth's shadow. For fields left blank the values defined in

Figure 15.2: Options for defining the setup of stochastic orbit parameters in GPSEST.

the above options are used. Entering a value overwrites the default values. In order to setup pulses for all satellites you need to specify a 99-line.

Enabling the checkbox "Additional requests for further epochs" activates still another panel where you get the possibility to specify in detail also the pulse epochs individually for each satellite. These stochastic pulses are set up in addition to the pulses defined in panel "GPSEST 6.8.2". Bear in mind that pulse epochs are shifted to the nearest integration subinterval boundary.

After having executed program GPSEST with observations covering one day, you have (among many other result files) an orbital element file, a normal equation file, and an Earth rotation parameter file at your disposal. Keep in mind that in general you should also estimate Earth rotation parameters when you determine the satellite orbits. More information on this topic may be found in Section 15.4.

15.3.1.3 Update Improved Orbit

Updating of a standard orbit using program ORBGEN according to an orbital element file is the last step of an orbit improvement. Insert the orbital element file as well as the Earth rotation parameter file generated in the previous step with program GPSEST (or alternatively with program ADDNEQ2, see Section 15.3.2) to obtain an updated standard orbit file. You may select two element files which will then be merged into a single standard orbit file (e.g., separate element files for GPS and GLONASS).

You may define the length of the arc interval (starting at the osculation epoch, i.e., the start epoch of the a priori orbit) in the program panel. You may also specify whether you actually want to use the updated orbits or if you want to reconstruct the a priori orbit which was used for the orbit

improvement (option OLD resp NEW in the field "SELECTION OF ORBITAL ELEMENTS" in panel "ORBGEN 4: Selection of Orbital Elements").

It is essential that the orbit models used for the generation of the a priori orbit and for updating of the orbit are identical. Program ORBGEN makes sure that, e.g., the same gravity field or satellite information file is used. The update process also takes the estimated stochastic pulses into account which are (also) stored in the orbital element files. The updated standard orbit may be converted to a precise orbit file using program STDPRE as described in Section 15.2.2.
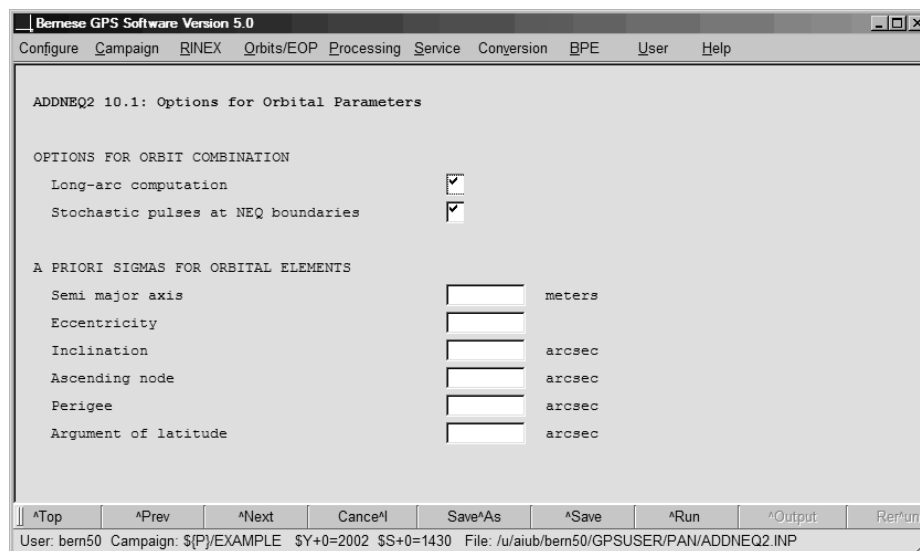
In order to perform an additional orbit improvement iteration, write a radiation pressure file and repeat the step described in Section 15.3.1.2 using the updated orbit information as well as the update step described in this section.

15.3.2   Orbit Combination with ADDNEQ2

Orbital parameters, if saved in normal equation files, can be manipulated by program ADDNEQ2. It is in particular possible to change the constraining of parameters or to combine, e.g., one-day arcs to longer arcs. The theory for this longarc combination may be found in [*Beutler et al.*, 1996].
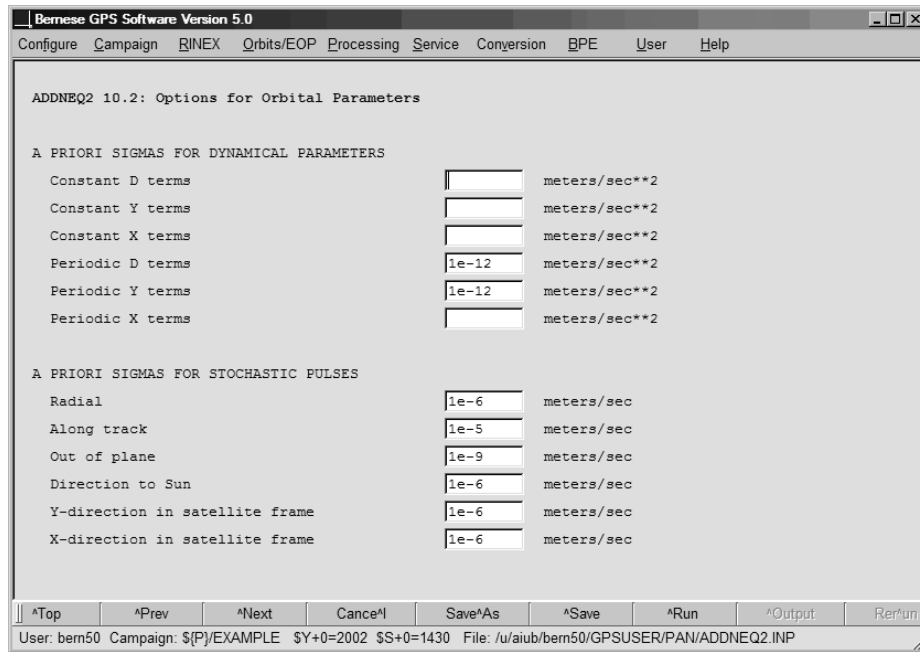
Manipulation of orbital elements in ADDNEQ2 is initiated by supplying one or several NEQ files that contain orbital parameters and to enable option "Orbital parameters" in panel "ADDNEQ2 3.2: Options 2". Additional panels are then activated that allow for the definition of orbit-specific options. Figures 15.3 and 15.4 show the panels designed for specifying the constraints for osculating elements as well as of dynamical and pseudo-stochastic parameters.

Option "Long-arc computation" in the first panel enables long-arc combination of arcs if a sequence of consecutive NEQ files is provided as input that contain orbital parameters. The output is then a series of orbital element files. In addition, new pseudo-stochastic parameters may be set up at the NEQ boundaries with option "Stochastic pulses at NEQ boundaries". Combination of arcs at particular

Figure 15.3: Panel for defining orbit estimation options in ADDNEQ2.

Figure 15.4: Options for defining constraints for dynamic and stochastic parameters in ADDNEQ2.

NEQ boundaries for specific satellites (e.g., for badly modelable orbits) may be suppressed by supplying a satellite problem file containing corresponding splitting information (see Section 22.4.6) as input file to ADDNEQ2.

If normal equation files of different sessions are combined (e.g., to compute long arcs), the output of program ADDNEQ2 is a sequence of orbital element files, one for each input NEQ. In this case you have to use wildcard characters (either % or ?) in the output file name in the field "Orbital elements". The wildcard characters are then replaced by the corresponding characters of the a priori standard orbit file name used to create the normal equation files. To give an example, let us assume that the a priori standard orbit files used to generate three 1-day normal equation files were named G1_06061.STD, G1_06062.STD, and G1_06063.STD and that you would like to create a 3-day arc with program ADDNEQ2. If you specify G3_%%%% for the orbital element output file name, ADDNEQ2 will save the estimated orbital parameter for each day in the files G3_06061.ELE, G3_06062.ELE, and G3_06063.ELE. Be in particular careful when processing several sessions with the BPE in order not to overwrite element files from older by files from newer sessions.

The orbital element files obtained as output from ADDNEQ2 may be updated using program ORBGEN as described in Section 15.3.1.3. The resulting standard orbits may finally be converted to precise files (see Section 15.2.2).

15.3.3 Production of Three-day Orbial Arcs at the CODE Analysis Center

In order to demonstrate that the actual work at an orbit determination center may be quite involved (mainly due to the fact that the data have to be screened and validated), we briefly present the actual procedure to generate a three-day arc at the CODE Analysis Center:

- The a priori orbit is taken over from the generation of the CODE Rapid Orbit.

- All single-difference files of the day are processed in GPSEST by modeling the correlations correctly on the baseline level, only. Orbits are improved with stochastic parameters tightly constrained.

- The residuals with respect to this "first" one-day arc are checked for outliers, bad phase observations are marked.

- With these screened observation files, GPSEST is invoked again, still in the single baseline mode, but this time stochastic orbit parameters are estimated at noon.

- The new one-day arc is used to resolve ambiguities on the single baseline level.

- After ambiguity resolution, a new (already very precise) a priori orbit without stochastic parameters is defined and used for the remaining one-day solutions. This orbit is then also the basis for all three-day solutions.

- Again, we use GPSEST, and again we set up all 15 orbit parameters, but we tightly constrain all of them. Moreover, we do not process the entire one-day data set in one program run, but we produce four regional cluster solutions, where within each cluster (corresponding to one run of program GPSEST) the correlations are modeled correctly, and the ambiguities resolved previously are introduced as known.

- The four cluster solutions are superposed using program ADDNEQ2 to give the final one-day solution.

- Three consecutive NEQ files corresponding to the final one-day solutions are combined to yield the final three-day solution.

- Any number of alternative solution series may now be produced using ADDNEQ2, only.

15.4   Estimation of Earth Orientation and Geocenter Parameters

15.4.1   Motivation

By the term Earth Orientation Parameter (EOP) we understand a set of five parameters describing the orientation of the ITRF (International Terrestrial Reference Frame) in the ICRF (International Celestial Reference Frame) in conjunction with the conventional Precession-Nutation model. The parameters x_p, y_p locate the Celestial Intermediate Pole (CIP) (formerly called Celestial Ephemeris Pole, CEP) in the terrestrial reference frame, whereas the position of the CIP in inertial space is defined by the IAU 2000 Theory of Nutation. The parameters $d\psi$ and $d\varepsilon$ reflect the difference between the actual position of the celestial pole and the position given by the conventional IAU model. Finally, the difference UT1–UTC (respectively UT1–TAI, TAI=International Atomic Time) gives access to the direction of the IERS reference meridian in the celestial reference frame (for details we refer to [*Castrique*, 1996]).

Station positions are conveniently represented in an Earth-fixed reference frame rotating in a well-defined way with the Earth. This reference frame is realized today by a catalogue of adopted geocentric coordinates and a velocity field of tracking stations derived from the analysis of data from various space techniques (VLBI, SLR, GNSS, DORIS). The frame is designated as the International Terrestrial Reference Frame (ITRF) maintained by the International Earth Rotation and Reference Systems Service (IERS).

The motion of the GNSS satellites is best described in the Inertial Celestial Reference Frame (ICRF), realized by a list of adopted equatorial coordinates (epoch J2000.0) of extra-galactic radio sources uniformly distributed over the sky.

In order to compute the difference vector between the observing station and the satellite both positions must be given in the same reference frame. Therefore, we need to know the current set of transformation parameters (EOP) between the two frames when analyzing GNSS data. Moreover, when processing data from a global GNSS tracking network, it is possible to estimate a subset of EOP parameters if the positions of some tracking sites are known in the ITRF.

The satellite's equations of motion are formulated in a reference frame with a particular origin that may be identified with the center of mass of the total Earth. On average this center of mass coincides with the origin of the ITRF. Variations of the geocenter coordinates (position of the Earth's center of mass with respect to the ITRF origin), e.g., due to changing mass distribution in the oceans and the atmosphere may be measured with satellite geodetic methods.

15.4.2   Theory

The transformation between the celestial and the Earth-fixed coordinate system may be performed by means of equation

$$ \boldsymbol{r}_E = \boldsymbol{X}^\top \boldsymbol{Y}^\top \boldsymbol{S} \ \boldsymbol{N} \ \boldsymbol{P} \ \boldsymbol{r}_I \tag{15.1} $$

where $\boldsymbol{r}_E$ and $\boldsymbol{r}_I$ denote the position vectors of a station in the terrestrial and inertial systems, respectively. The sequence of rotation matrices $\boldsymbol{N} \ \boldsymbol{P}$ referring to nutation and precession describes the transformation between the mean celestial system at epoch J2000.0 and a system defined by the true equator and equinox of date. The *Bernese GPS Software*, Version 5.0, supports both IAU80 and IAU2000 nutation theories. The matrix $\boldsymbol{S} = \boldsymbol{R}_3(\Theta_{GA})$ provides the transition to the rotating system where Θ_{GA} is the Greenwich apparent sidereal time ($\boldsymbol{R}_i(\alpha)$ characterizes a rotation around axis i, about angle α). Finally, the polar motion matrices $\boldsymbol{X} = \boldsymbol{R}_2(x_p)$ and $\boldsymbol{Y} = \boldsymbol{R}_1(y_p)$ describe the position of the Celestial Intermediate Pole (CIP) in the Terrestrial Reference Frame.

To the accuracy level required for the computation of the partial derivatives of the GNSS observable with respect to the parameters of interest, we may approximate the nutation matrix as a product of three infinitesimal rotations and write the transformation equation (15.1) in the form

$$ \boldsymbol{r}_I = \boldsymbol{P}^\top(t) \ \boldsymbol{R}_1(\Delta\varepsilon) \ \boldsymbol{R}_2(-\Delta\psi \sin\varepsilon_0) \ \boldsymbol{R}_3(-\Theta_{GM}) \ \boldsymbol{R}_1(y_p) \ \boldsymbol{R}_2(x_p) \ \boldsymbol{r}_E \ , \tag{15.2} $$

where $\Delta\psi$ and $\Delta\varepsilon$ denote the nutation in longitude and obliquity, ε_0 denotes the mean obliquity of the ecliptic, and Θ_{GM} stands for the Greenwich mean sidereal time. All five Earth orientation parameters are contained in Eqn. (15.2).

Unfortunately, due to correlations with the orbital elements, a subset of the EOP is not directly accessible to the GNSS (namely $\Delta UT = UT1 - UTC$ and the nutation parameters). It is, e.g., possible to add a constant value to UT1 and adapt the ascending nodes of the satellite orbits by a corresponding value without affecting the ranges between stations and satellites. On the other hand, it is possible to solve for a drift in UT1-UTC allowing to estimate the length of day (LOD) very well with the GNSS. Similarly drifts in the nutation parameters are accessible by GNSS [*Rothacher et al.*, 1998].

15.4.3 Estimation of Earth Orientation Parameters

Today, in view of the products made available through the IERS and the IGS it will in general not be necessary to solve for EOPs, but to use EOPs consistent with the orbits and coordinates used. For research type applications based on data stemming from a global GNSS network it makes sense to set up EOPs. This section is important for advanced users of the *Bernese GPS Software* who wish to gain insight into global applications of GNSS. Let us start by mentioning a few basic facts:

- You will always estimate ERPs on top of an a priori model. These are characterized by the values in a pole file (default extension ERP). If you want to estimate the ERPs from scratch you have to use an a priori file with all parameters set to zero.

- The (empirical) model for the ERPs is set up in program GPSEST, it may be modified in program ADDNEQ2, where you have to be aware of the fact that you may simplify but not generalize the model in ADDNEQ2.

- If you solve for ERPs you will probably process exactly one session of one day in GPSEST and produce longer arcs using ADDNEQ2. You should be aware of the fact that for the ERP estimates you may divide the time interval of the session into a number of subintervals of equal length. In program GPSEST each ERP (improvement on top of the a priori model) is represented by a polynomial within each subinterval while in ADDNEQ2 the ERP improvements are represented by a piece-wise linear function with nodes at the subinterval boundaries (see Section 7.5).

- For diurnal and semi-diurnal variations of the ERPs, the IERS 2003 model ([*McCarthy and Petit*, 2004]) or the model from [*Ray et al.*, 1994] may be used. The model name is logged in the pole file header.

- UT1-UTC estimates are special in one respect: due to the correlations with the orbital nodes mentioned above, it is not possible to solve for the zero degree coefficient of the polynomial pertaining to the first sub-interval of the session (GPSEST) or the first value in the piece-wise linear representation (ADDNEQ2) (if you introduce the orbital elements as unknowns in the same adjustment). The same is the case for nutation parameters.

15.4.3.1 Options in GPSEST

Panel "GPSEST 5.2: Setup of Parameters and Pre-Elimination 2" handles many special requests in connection with parameter estimation. You have to enable the parameter type "Earth orientation parameters". In this case the menu system gives you access to panel "GPSEST 6.13: Earth Orientation Parameters" (see Figure 15.5), where you may first define the spacing of the Earth orientation parameter sets (=total number of polynomials for each EOP). The start and end time of the parameter sets are computed using the general parameter time offset defined in option "TIME OFFSET FOR PARAMETER INTERVALS" in panel "GPSEST 5.2".

Subsequently you may select the number n of polynomial coefficients for each of the five EOPs in the fields "Par/set". The polynomial degree q is simply $q = n - 1$. $n = 0$ means that the corresponding parameters are not set up. You have to be aware of the fact that your estimates will in general not be continuous at the sub-interval boundaries. You may ask for continuity for the pole components x_p, y_p, and for UT1-UTC by specifying "ERP" in the field "Continuity between sets". If

you want to enforce continuity for the two nutation parameters you specify "NUT", and you specify "BOTH" if you want to have continuity for all parameters. Attention: If you ask for continuity in the case $q = 0$ ($n = 1$), this actually means that you model the EOPs by a single parameter for the entire session! If you ask for continuity in the case $q = 1$ ($n = 2$), you actually model the EOPs as polygons (standard procedure at CODE). We strongly recommend to use $q = 1$ ($n = 2$) and to ask for continuity. Only EOP values represented by offsets and drifts ($n = 2$) can be properly handled by program ADDNEQ2.

You may constrain the first-degree polynomial coefficients of the ERP, the nutation, or both parameter sets to zero by activating the option "Drifts constrained to zero". This makes sense if you intend to estimate drift parameters later with program ADDNEQ2 using more than one session.

The zero-degree coefficients of each parameter set may be constrained independently for each of the five EOP parameter types by specifying an a priori sigma in the fields "Other parameters". Because, as mentioned above, it is not possible to solve for all orbit parameters and for UT1-UTC or nutation parameters, the zero-degree parameters pertaining to the first sub-intervals can be constrained separately for these parameters with a priori sigma values in the fields "First parameter".

If you define the EOPs according to the panel displayed in Figure 15.5, GPSEST will generate an output (in the general OUTPUT file) of the following type:

```
EARTH ROTATION PARAMETERS:
--------------------------

CRD. REQ.    (")         RMS        (")/DAY      RMS       (")/DAY**2
             (S) (DT)    RMS        (S)/DAY      RMS       (S)/DAY**2
---------------------------------------------------------------------

 X    1    0.0010080  0.0000900    -0.0007370  0.0001536
 Y    1    0.0017345  0.0001013    -0.0013315  0.0001621
 DT   1    0.0000000  0.0000000    -0.0002006  0.0000064
```

The spacing in time of the ERP values saved in the output Bernese ERP file and the IERS ERP file may be specified in option "TIME RESOLUTION OF EXTRACTED VALUES", independently of the parameter spacing used.

15.4.3.2 Options in ADDNEQ2

With program ADDNEQ2 you may combine normal equation systems generated by GPSEST (or ADDNEQ2). The program allows you to reconsider some aspects of EOP estimation. EOPs estimated by GPSEST are parameterized as polynomials within sub-intervals while ADDNEQ2 uses a piece-wise linear parameterization. ADDNEQ2 automatically converts EOP parameters represented as offset and drift (polynomial of first degree) into the piece-wise linear representation if an interval length is specified in the section "PARAMETER SPACING" in panel "ADDNEQ2 8: Interval Length of Parameters" (see Section 9.4.5) which is strongly recommended. You may specify the same or a longer interval (e.g., 24 00 00) than the one you originally defined in GPSEST when setting up EOP parameters.

Enable the checkbox "Earth orientation parameters" in panel "ADDNEQ2 3.2: Options 2" in order to get the EOP-specific panel "ADDNEQ2 11: Options for Earth Orientation Parameters" (see Figure 15.6). The options in this panel resemble those in program GPSEST: You may define a priori
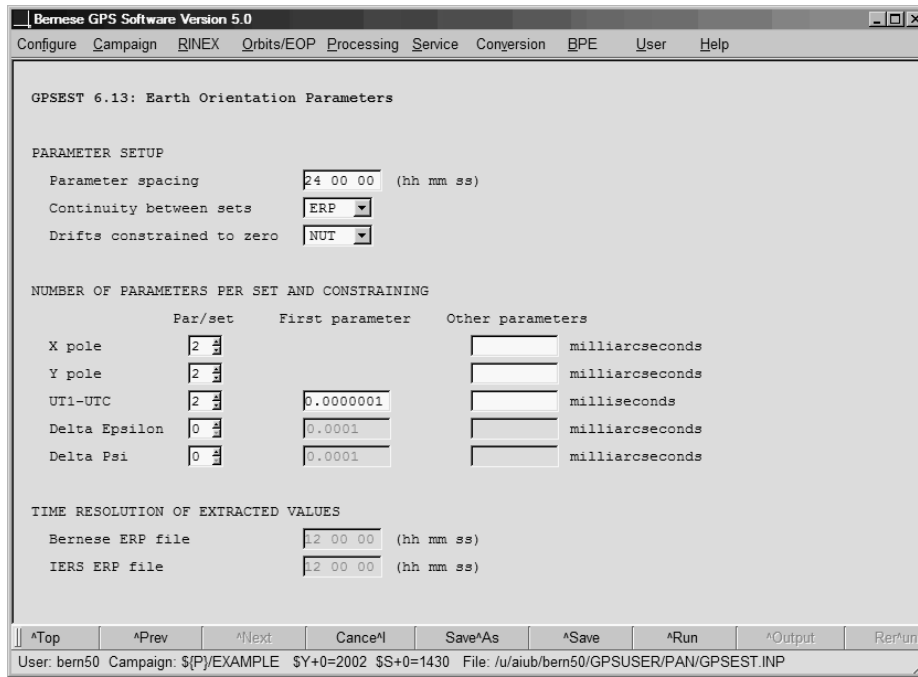
Figure 15.5: Options for defining the setup for ERP parameters in GPSEST.
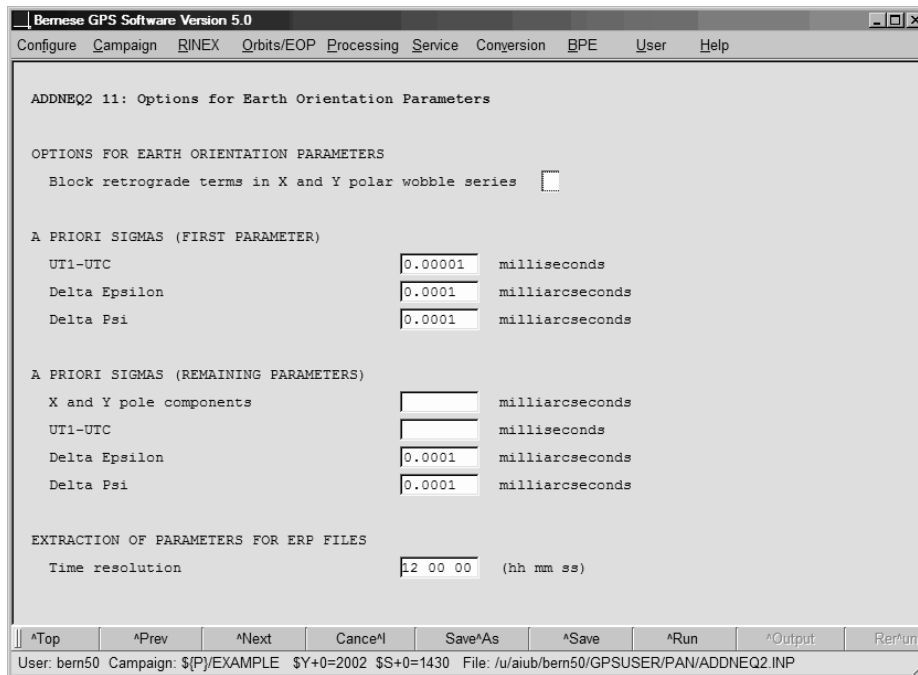

Figure 15.6: Options for defining the setup for ERP parameters in ADDNEQ2.

constraints for the first offset parameters for UT1-UTC and the nutation parameters as well as for all remaining offset parameters for all parameter types.

The option "Block retrograde terms in X and Y polar wobble series" is only of importance if you are interested in a sub-daily resolution of the EOPs in order to cope with correlations of daily retrograde motion of the pole with the orientation of the satellite's orbital planes [*Hefty et al.*, 2000].

There is one more important difference between the EOP estimation in GPSEST and ADDNEQ2: Whereas in GPSEST the corrections to the a priori pole are modeled as polynomials, the absolute values and UT1R (obtained from UT1 by removing the tidal variations with periods < 35 days) are represented as piece-wise linear functions in ADDNEQ2.

15.4.4 Estimation of Geocenter Coordinates

Because satellites are sensitive to the center of mass of the total Earth (geocenter), satellite geodetic techniques are in principle sensitive to motions of the center of mass with respect to the origin of a crust-fixed reference frame. Such motions are expected due to mass displacements in the oceans, the atmosphere, and the interior of the Earth as well as from surface deformations originating from loading effects.

In the *Bernese GPS Software*, Version 5.0, geocenter coordinates may be estimated in programs GPSEST and ADDNEQ2. It has, however, to be remarked that the sensitivity of GNSS to geocenter motions is low due to correlations of these parameters with radiation pressure modeling parameters.

In program GPSEST the estimation of geocenter coordinates is initiated by enabling the corresponding checkbox "Geocenter coordinates" in panel "GPSEST 5.2: Setup of Parameters and Pre-Elimination 2". The menu system then displays panel "GPSES 6.14: Geocenter Coordinates" which allows to switch on and to constrain the estimation for each coordinate component individually. Geocenter coordinates are modeled in the Earth-fixed frame as constant values for the entire session.

Program GPSEST does not write a geocenter output file but includes the parameters into the output NEQ file for further analysis with ADDNEQ2 and generates the following output in the general program output file (values in meters):

```
CENTER OF MASS:
--------------

COOR.       VALUE        RMS
------------------------------------------------------------------

  X          0.0131         0.0009
  Y          0.0026         0.0010
  Z          0.0088         0.0013
```

Program ADDNEQ2 gets information concerning geocenter coordinates through the input NEQ files. The parameters may be deleted, constrained, or pre-eliminated. A priori constraints may be defined in panel "ADDNEQ2 12: Options for Additional Parameters".

Since geocenter coordinates may be looked at as a common translation of the reference coordinates, the estimation of geocenter coordinates may be initiated in ADDNEQ2 even if the input NEQs do not contain this parameter type. The corresponding option "Set up geocenter coordinates" may be enabled in panel "ADDNEQ2 3.1: Options 1". The option has no effect if the input NEQs already contain geocenter coordinates.

Estimated geocenter coordinates may be written to an output file (default extension GCC, see Section 22.7.9) by specifying a filename in the field "Geocenter coordinates" in panel "ADDNEQ2 2: Output Files". But also a geocenter input file may be specified in panel "ADDNEQ2 1: Input Files" allowing to transform a priori values from NEQ files to external values.