GPS SATELLITES: RADIATION PRESSURE, ATTITUDE AND RESONANCE

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ABSTRACT

At the altitude of the GPS satellites the most important non-gravitational perturbation is caused by the solar radiation pressure acting on the satellite body and its solar panels. The development of high-fidelity radiation pressure models may be motivated by the following observation: The GPS satellites are orbiting in a 2:1-commensurability with the Earth's rotation which causes resonance. The expected sensitivity to specific coefficients of the geopotential is, however, significantly reduced by strong correlations of these parameters with radiation pressure parameters. Sophisticated radiation pressure models rely on a precise knowledge of the satellite's attitude which does not only affects the location of the antenna phase center or the phase windup of the signal carrier but, through radiation pressure, also the orbital dynamics. PRN 23, whose attitudinal behaviour was modified early in 2002 is an interesting case. Due to this change an impressive improvement in the orbit quality could be achieved.

INTRODUCTION

The Global Positioning System (GPS) is a satellite navigation system operated by the Department of Defense. Its acronym is today used as a synonym for precise positioning and navigation. Starting with the first launch of a satellite of the NAVSTAR GPS series on February 2, 1978, the signals emitted by the satellites have put new standards for applications ranging from navigation to highest precision geodetic applications. The space segment consists of currently 28 active satellites distributed in six orbital planes, separated by 60° in the ascending node; the orbital inclinations with respect to the equatorial plane are close to 55°. The nearly circular orbits – the eccentricities range from 0.001 to 0.016 – have a semimajor axis of 26'560km or 4.16 Earth radii, which places the satellites at an altitude of about 20'000km above the Earth's surface. The orbital period is almost exactly half a sidereal day, $11^{\mathrm{h}}56^{\mathrm{m}}$, which guarantees a ground track repeatability within one sidereal day.

Towards the end of the 1980s the necessity for a high-precision GPS orbit determination service for scientific applications became evident. This is why the International GPS Service was created. An IGS Test Campaign was started on June 21, 1992, with the goal to demonstrate that routine computations of precise GPS satellite orbits is possible based on the available tracking network which consisted at that time of about 40 globally distributed receivers (Beutler et al., 1999). In 1993 the International Association for Geodesy (IAG) approved the IGS as one of its services, and the IGS started routine operations in January 1994. It turned out to be one of the big successes in worldwide scientific collaboration. Today, more than 330 tracking stations are in service worldwide, operated by a multitude of different institutions making their data publicly available within one day, one hour, or even in realtime for the scientific community (see the IGS web page at http://igscb.jpl.nasa.gov).

Seven Analysis Centers (AC), one of them CODE (Center for Orbit Determination in Europe) located at the Astronomical Institute of the University of Bern, compute high precision GPS satellite orbits as well as satellite and receiver clock corrections, Earth orientation parameters, station coordinates and velocities,

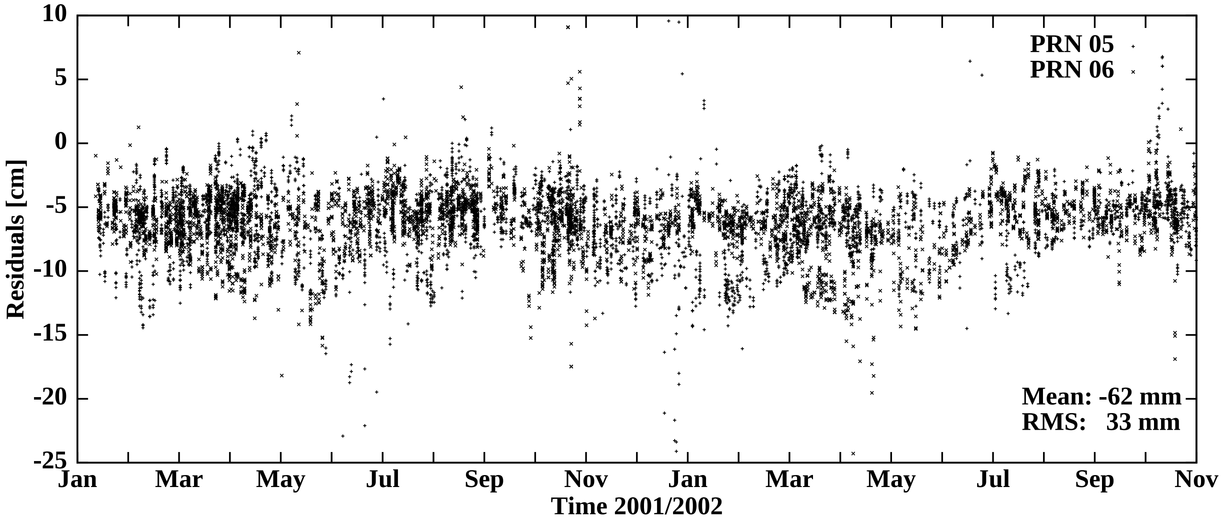

Fig. 1. SLR residuals for the two GPS satellites PRN 5 and 6 equipped with retroreflectors. Residuals are understood as SLR measurements minus range obtained using IGS final orbits and ITRF2000 site coordinates.

troposphere and ionosphere parameters. Different software, models, and analysis strategies guarantee an independence of solutions as well as a fruitful competition between the analysis groups.

The orbits stemming from the different ACs are combined by the IGS Analysis Center Coordinator (ACC) and result in the official IGS orbits. The consistency between the best IGS contributions has reached a level of only about three centimeters. External validation of the IGS orbits using Satellite Laser Ranging (SLR) observations to the two GPS satellites PRN 5 and PRN 6 equipped with Laser retroreflectors confirm an accuracy around 5cm. Figure 1 shows SLR residuals for the time period from January 2001 to October 2002. The residuals reveal a systematic bias of -6.2 ± 3.3cm which was already identified by Springer (2000) and others. It is still unclear why the measured SLR distances are shorter than the distances emerging from the GPS orbits derived from microwave observations. Systematic variations with time containing a signature characteristic for radiation pressure may indicate a slightly inconsistent scale as determined by the microwave technique. The clarification of the differences between SLR and microwave need further investigations.

RESONANCES DUE TO THE GEOPOTENTIAL

Due to the almost perfect 2:1-commensurability of their orbital period with the Earth rotation the GPS satellites are in resonance with the Earth's gravitational potential. The most important coefficients in the harmonic expansion of the potential leading to this resonance are the 32-term followed by the 44-term. The resonance induces a daily drift of up to 7m/day in the semimajor axis leading to a very longperiodic perturbation with an amplitude up to 6km and a period of 8 years or more (Hugentobler, 1998).

As a consequence of this resonance station keeping manoeuvres are necessary in regular intervals (about once a year) in order to keep the satellites at their nominal position in the orbital plane. GLONASS satellites orbiting at a roughly 1000km lower altitude perform $2^1/_8$ revolutions per sidereal day. They are, therefore, only in a shallow 2:1-resonance and need no manoeuvres due to this resonance. The resonance makes GPS satellites, however, particularly sensitive to the resonant terms of the geopotential. Tracking data may, therefore, be used to estimate these coefficients.

Figure 2 shows the a priori rms for the resonant term S_{32} computed on the basis of an orbital arc of one day (left) and seven days (right) for each GPS satellite and, for comparison, for each active GLONASS satellite. The a priori rms was determined based on a covariance analysis (for details we refer to Ineichen et al. (2003)). It is interesting to note that the GPS satellites do not behave as expected. The rms of the resonant coefficient obtained from the GPS satellites decreases less rapidly with increasing arc length than in the case of GLONASS satellites. Correlations with radiation pressure parameters (estimated together

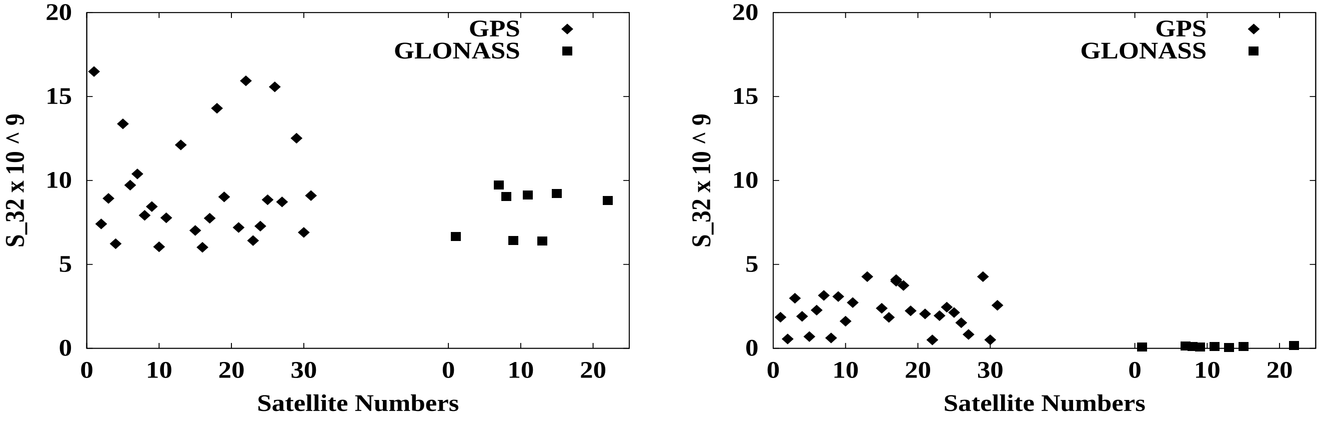

Fig. 2. RMS a priori for the S_{22} coefficient estimated individually for each GPS and GLONASS satellite based on a 1-day arc (left) and a 7-day arc (right).

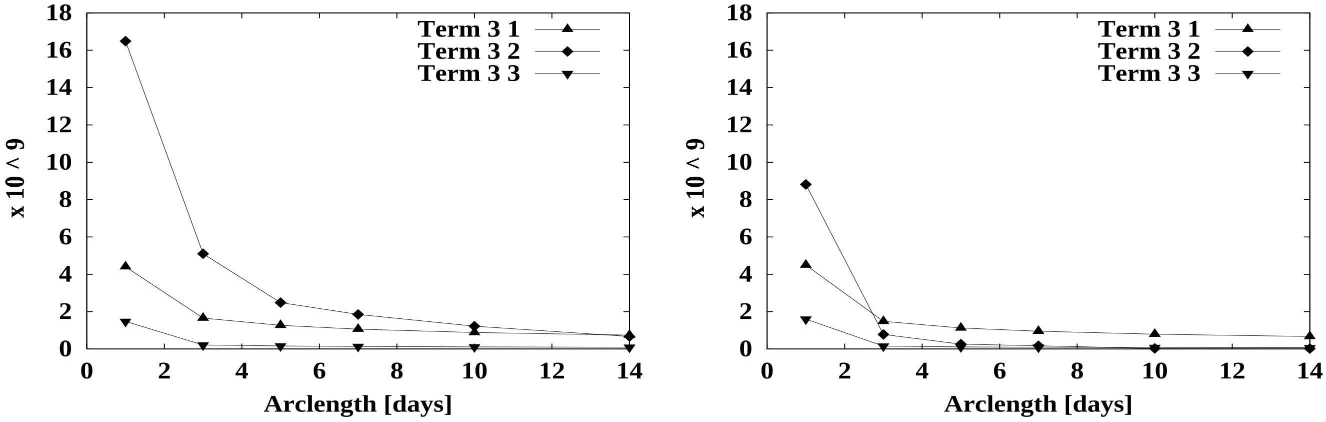

Fig. 3. Dependence of a priori rms for selected sine geopotential coefficients on arclength for GPS satellite 1 (left) and GLONASS satellite 4 (right).

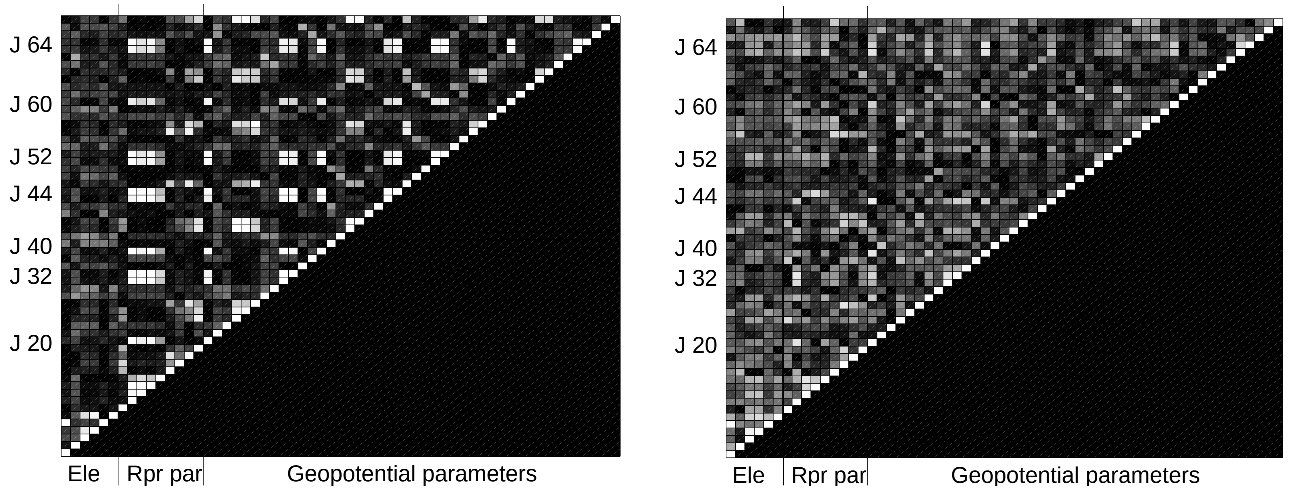

Fig. 4. Correlation matrix (upper triangle) between orbital elements, radiation pressure parameters, and geopotential coefficients estimated using GPS satellite 4 (left) and GLONASS satellite 1 (right).

with the geopotential coefficients) are responsible for this behaviour. In fact, some of the radiation pressure parameters induce similar perturbation characteristics than the resonant geopotential terms.

Figure 3 (left) shows that the a priori rms for the resonant term S_{32} improves only slightly faster with increasing arclength than nonresonant terms. The improvement with the arclength may be described by a power law with an exponent close to $-1/2$. For GLONASS satellites, on the other hand (see right hand panel in the figure), the improvement of the sensitivity to S_{32} with arclength is close to a power law with exponent $-3/2$ as expected for a resonant parameter. If no radiation pressure parameters are estimated simultaneously with the gravity potential parameters, the a priori rms of the resonant coefficients for GPS behaves as that for GLONASS. This is a clear indication that correlations between radiation pressure and geopotential parameters in deep resonance hides the sensitivity with respect to resonant coefficients for GPS satellites.

Figure 4 shows the upper triangular part of the correlation matrix for the parameters estimated using GPS satellite 4 (left) and GLONASS satellite 1 (right) based on a 1-day arc. White areas indicate a high correlation while dark areas indicate a low correlation. A prominent pattern of correlations between radiation pressure and geopotential parameters as well as among different potential coefficients can be observed for the GPS satellite while no distinct pattern is present in the case of the GLONASS satellite. The correlations obviously are amplified by the commensurability of orbital revolution of the satellites and sidereal rotation of the Earth. GPS satellites are thus less well suited to trace resonant geopotential coefficients than GLONASS satellites, as long as these parameters are estimated together with radiation pressure coefficients, which is unfortunately mandatory. The use of high-fidelity radiation pressure models may help to improve this situation for GPS satellites.

GPS SATELLITE ATTITUDE

Radiation pressure models require the knowledge of the satellite's attitude. The use of the correct attitude for the GPS satellites is important in order to (a) correctly account for dynamic effects – e.g., due to a misalignment of the solar panels to the Sun –, (b) to compute the location of the antenna phase center, and (c) to correct for the phase windup affecting phase measurements of circularly polarized radiation. While the phase windup due to attitude motion of the satellite is absorbed by clock corrections, an erroneous antenna phase center and a satellite misalignment with respect to the Sun definitely affect the quality of the estimated orbits.

Nominally the z-axis of the body fixed coordinate system is always pointing to the center of the Earth while the y-axis is parallel to the axis of the solar panels and is perpendicular to the direction of the incident sunlight. The x-axis points into the hemisphere containing the Sun. The attitude is maintained by an autonomous attitude control system onboard the satellites which obtains information on spacecraft orientation from Sun and Earth sensors. The phase center offset for the antenna of Block IIA satellites is 28cm in the x-direction.

Attitude motion of the satellites involves rapid rotations if the Sun is close to the orbital plane. This includes so-called noon-turns (rotation close to the subsolar point) as well as rotations within the Earth's shadow and after shadow exit. Let us focus on Block IIA satellites. We study the eclipse of PRN 4 taking place over the Indian Ocean on doy 202/2002.

Figure 5 shows the zero-difference phase residuals exhibited by the observations of the satellite from Antarctic stations (left) and Asian stations (right) assuming a nominal attitude. The residuals show a distinctive pattern which is very similar for stations observing the satellite from the same direction, indicating that the satellite is not behaving according to nominal attitude.

Figure 6 shows double-difference residuals of the same event for a baseline crossing the ocean in north-south-direction (left) and in east-west-direction (right). A clear signature in and after the shadow can again be observed with respect to nominal attitude extending up to 20 minutes after returning into sunlight. If we do not assume nominal attitude but a rotation of the satellite around the z-axis (yaw-manoeuvre) at a maximum rate of 0.12°/sec (Bar-Sever, 1994), starting with shadow entry and ending with the reacquisition of nominal attitude, the systematic pattern in the residuals disappears. This is documented in Figure 7.

The clear signature of the residuals allows it to estimate the antenna phase center position in the x-y-plane with a high temporal resolution. The results for PRN 4 from three different days are displayed in

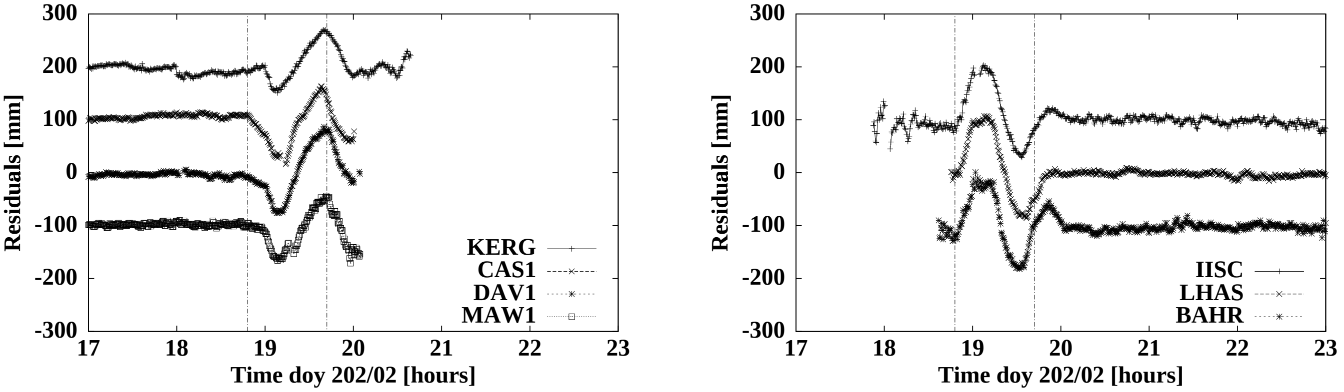

Fig. 5. Zero-difference residuals of observations of PRN 4 in eclipse from Antarctic stations (left) and Asian stations (right). Residuals from individual stations are shifted by 10cm. Vertical lines indicate start and end of eclipse.

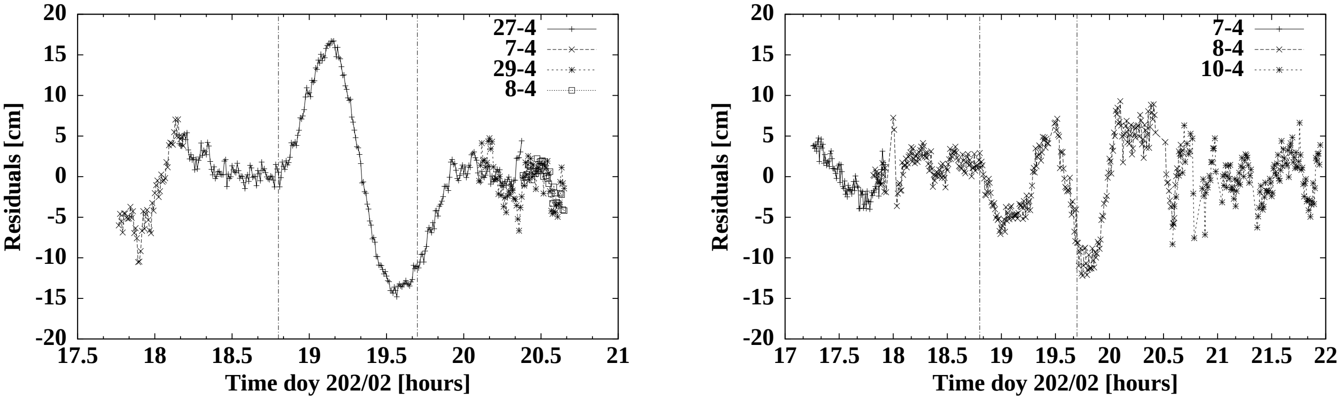

Fig. 6. Double-difference residuals for the baselines Kerguelen Islands (KERG)–Bangalore (IISC) in north-south-direction (left) and Karratha (KARR)–Mbarara (MBAR) in east-west-direction (right) assuming nominal satellite attitude. Different symbols refer to different reference satellites.

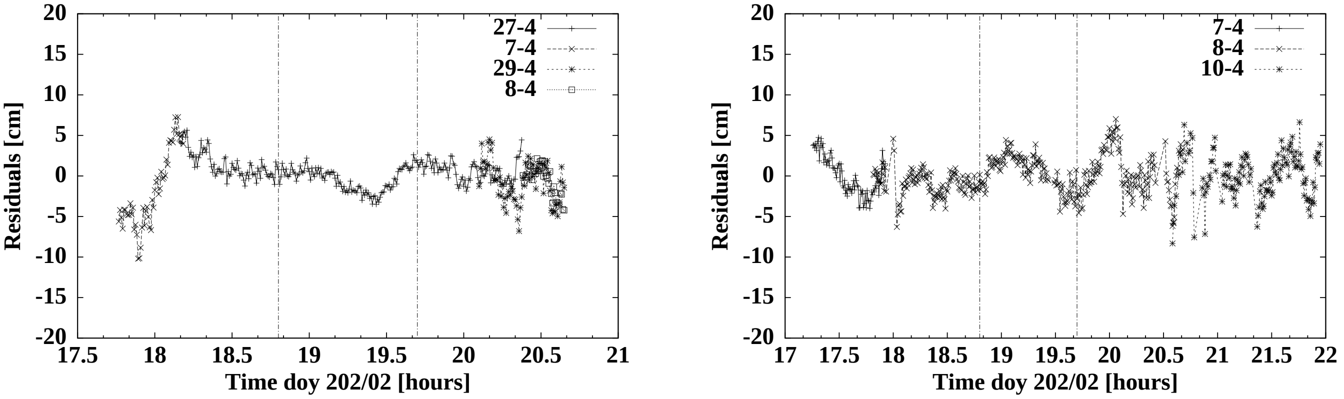

Fig. 7. Double-difference residuals for the same baselines as in Figure 6 assuming a yaw-rotation of the satellite with a maximum rate, starting with the entering into the shadow and ending with the reacquisition of the nominal attitude.

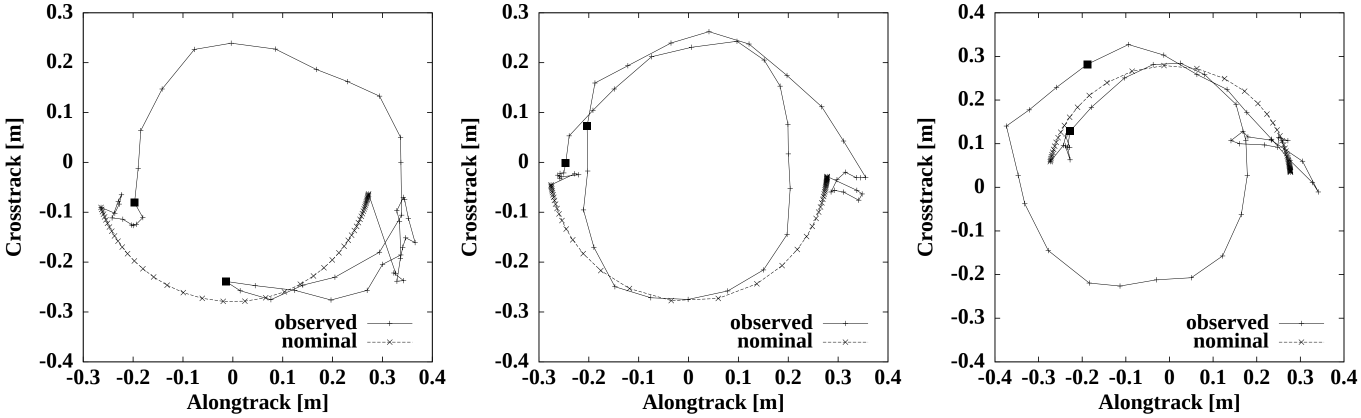

Fig. 8. Estimated antenna phase center position in the x-y-plane for PRN 4 for crossing of the Earth's shadow on three different days in 2002, doy 195 (left), doy 202 (center), doy 215 (right). Crosses give the positions according to nominal attitude, boxes indicate shadow entries and exits.

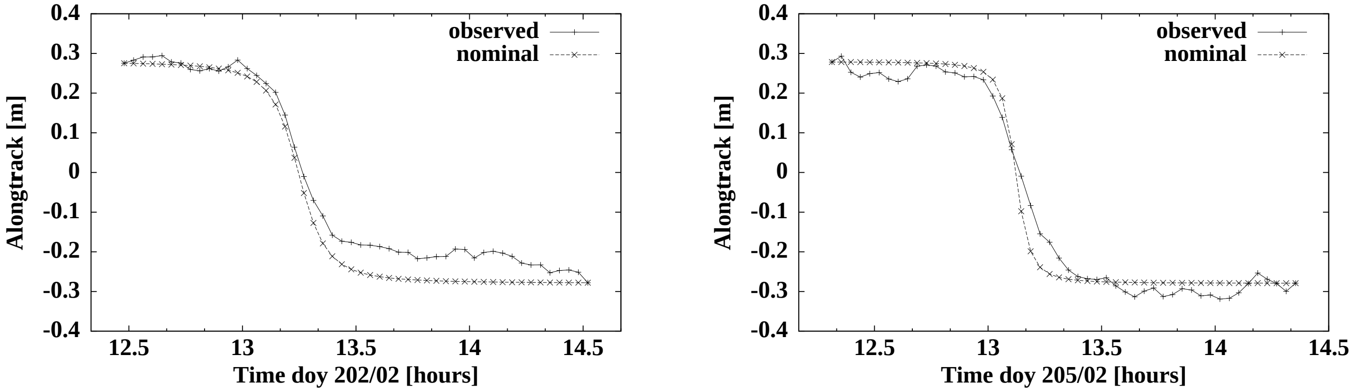

Fig. 9. Noon-turn of PRN 4 with a Sun elevation angle above the orbital plane of 4° on doy 202/2002 (left) and 2° on doy 205/2002 (right).

Figure 8. The phase center positions were estimated in intervals of 2.5 minutes within a time period of two hours covering the event of the eclipse. At the beginning and at the end of this 2-hour interval the position was constrained to the nominal position; within the interval relative constraints between successive antenna positions were imposed.

Figures 8 indicate an attitude manoeuvre which is far from nominal (indicated by crosses). On all three days the satellite started to rotate in the same direction with a yaw-rate of about 0.12°/sec when entering into shadow. On the first day the satellite reversed the rotation direction after shadow exit in order to reacquire nominal attitude as soon as possible. It is well known that Block II and Block IIA behave in the shadow as illustrated in the figures. Y. Bar-Sever developed – in close cooperation with the US Air Force – a sound understanding of the attitudinal motion of the GPS satellites (Bar-Sever, 1996). According to this information the nominal attitude of the Block IIA satellites is biased by a small yaw angle (rotation around z-axis) of 0.5°. This yaw-bias, which is kept constant for all satellites since fall 1995, enforces the satellites to start rotating in a well-defined direction as soon as they enter into the Earth's shadow. When returning into sunlight the satellite selects that rotation direction which brings it to nominal attitude in the shortest time.

Figure 9 shows two noon-turn manoeuvres of PRN 4 close to the subsolar point. In the left figure the elevation angle β above the orbital plane is 4° while in the right subfigure β is only 2°. It can be clearly seen

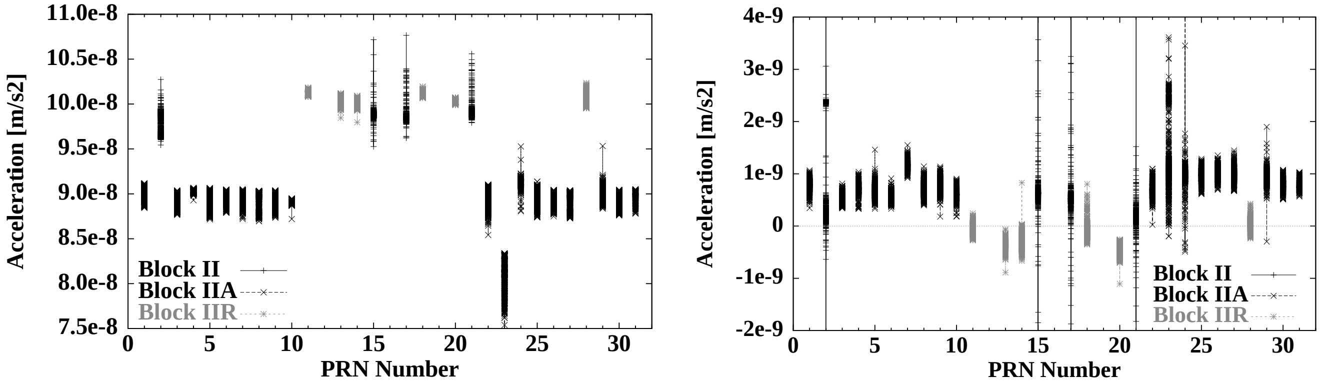

Fig. 10. Acceleration due to direct radiation pressure (left) and radiation pressure along the solar panel axis (right). Daily estimates for each satellite for the time period from January 2001 to October 2002.

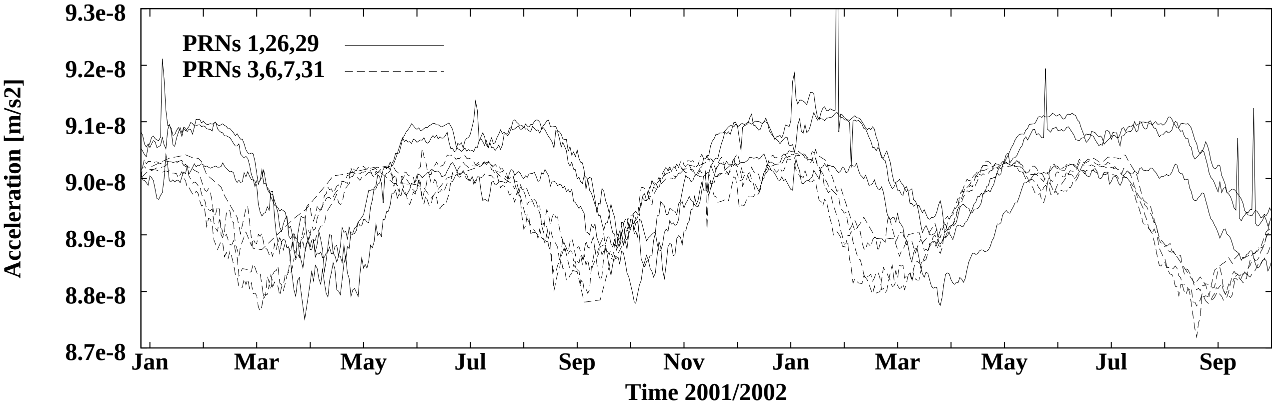

Fig. 11. Variation of the acceleration due to direct solar radiation pressure for Block IIA in two different orbital planes.

that in the latter case the yaw rotation of the satellite is slower than the nominal rotation rate requiring a maximum value of 0.23°/sec.

Block IIR satellites are much easier to model. They rotate according to nominal attitude in the shadow and do not perform a noon-turn if the Sun is closer than 1.6° to the orbital plane (Bar-Sever, 1997).

RADIATION PRESSURE

Due to the height of the GPS satellites radiation pressure represents the most important nongravitational force. The acceleration due to direct radiation pressure acting on solar panels and the satellite body is about $10^{-7}\mathrm{m/s^2}$, which is of a similar order of magnitude as the acceleration caused by the terms in the Earth's gravity potential of degree and order higher than oblateness. Satellite models developed by Rockwell (T20 and T30 models for Block II/IIA and Block IIR, respectively, Fliegel et al., 1992, Fliegel and Gallini, 1996) take into account the satellite's surface properties as well as thermal reradiation and provide the acceleration as a function of the direction of the incident solar radiation.

In addition to the deterministic model additional model parameters may be estimated. They are, e.g., included in the attitude model presented by Kuang et al. (1996) or in the Extended CODE Model (Beutler et al., 1996). The latter describes the accelerations in the direction to the Sun, the direction along the solar panel axis, and the third axis perpendicular to the two as a constant and a once-per-revolution periodic term. These nine model parameters are estimated for each satellite for every day using three day arcs at the CODE analysis center.

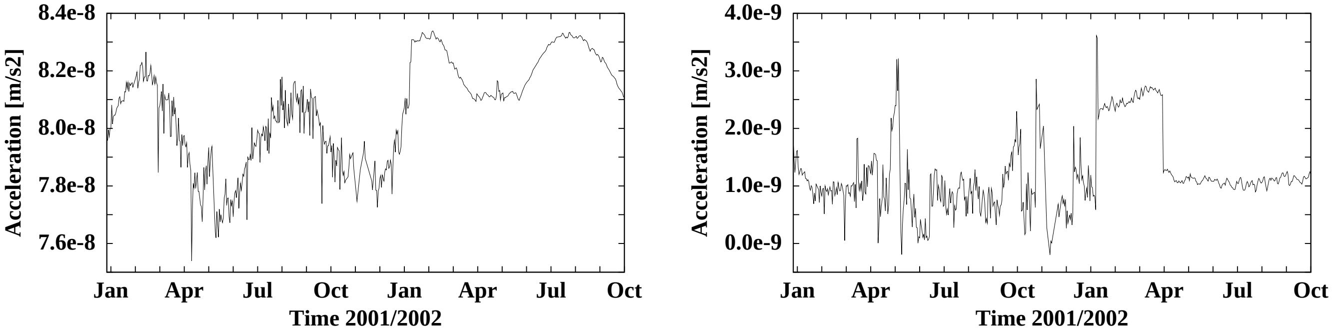

Fig. 12. Variation of the acceleration due to direct solar radiation (left) and y-bias (right) for PRN 23. Changes in the attitude strategy on January 12 and March 31, 2002, are evident.

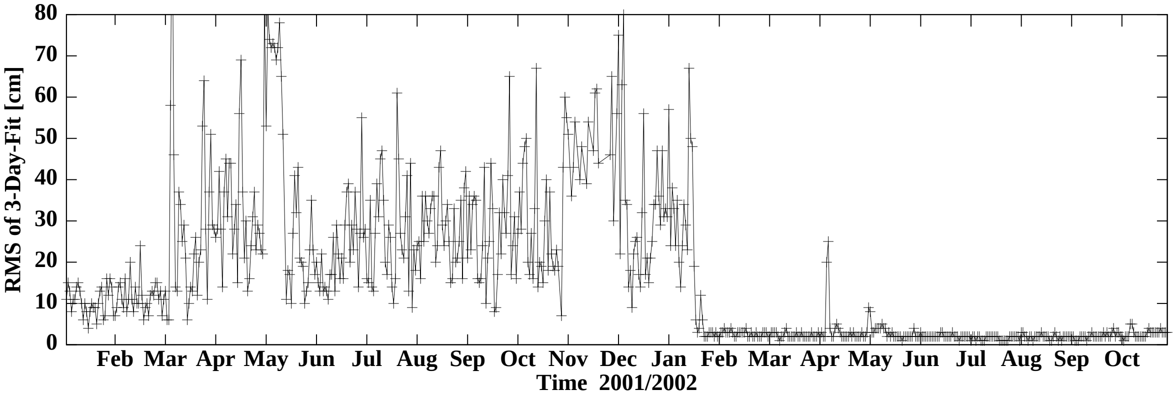

Fig. 13. RMS values of three-days fits of the orbit for PRN 23. The improved behavior of the satellite starting on January 11, 2002, is striking.

Figure 10 (left) shows the estimated daily values for the direct radiation pressure for each satellite in the time period from January 2001 to October 2002. It can be seen that Block II and Block IIR satellites experience a bigger average direct radiation acceleration than Block IIA. For the former this is due to their smaller mass, for the latter due to their larger surface area. The scatter of the values is due to semi-annual variations whereas the big scatter for Block II indicates frequent modeling problems. PRN 23 has a significantly lower acceleration due to the direct radiation pressure which is caused by a hardware problem with the solar panel array.

Figure 10 (right) shows the daily estimates for the acceleration along the solar panel axis for each satellite – the so-called y-bias. Clearly all Block IIA satellites have a positive y-bias, which may be due to the misalignment of the solar panels caused by the yaw bias of 0.5° superposed to the nominal attitude for the satellites of this type. The magnitude of the acceleration is, indeed, in rough agreement with the expected solar panel misalignment.

Variations of the direct radiation pressure acceleration show a semi-annual variation, which is common for satellites of the same type in the same orbital plane, and whose maxima coincide with the eclipsing seasons (see Figure 11). The amplitude of the variations depends on the range covered by the elevation angle of the Sun above the orbital plane. This range depends in turn on the location of the ascending node. Improvements of the Extended CODE Model, modeling these semi-annual variations, were implemented by Springer et al. (1999). Additional improvements of this model were added by Slabinski (2002).

Let us focus on PRN 23 which has a long-standing history of modeling problems. Figure 12 showing the time series of acceleration due to direct solar radiation (left) and y-bias (right) tells that on January 10 and on March 31, 2002, the behavior of the satellite is changed. Clearly the strategy for the attitude control was changed at these dates. The new strategy allows for a much improved representation of the satellite orbit. Figure 13 shows the rms of a series of 3-days fits of individual 1-day orbits of PRN 23. The improved orbital characteristics of the satellite starting end of January, 2002, is striking. Obviously we witness an in-orbit 'repair' of a GPS satellite. Before this date the solar panels were pitched manually, now they are pitched automatically (Bar-Sever, 2002).

CONCLUSIONS

Radiation pressure parameters strongly correlate with the resonant coefficients of the Earth's gravity field for satellites of the GPS. This correlation greatly reduces the sensitivity of the satellite system to these gravity parameters despite the fact that they are in deep resonance. For GLONASS satellites, on the other hand, the same geopotential coefficients do not cause very long periodic perturbations of the orbital parameters. Because the satellites are in shallow resonance, the induced periods are of the order of a few days which decorrelates resonant geopotential coefficients and radiation pressure parameters. GLONASS satellites are therefore better suited to study resonant gravity parameters than GPS satellites. To improve this situation for GPS satellites elaborate radiation pressure models may help.

About one percent of the acceleration due to solar radiation pressure experienced by the GPS satellites is acting in a direction other than that of the incident solar light. In order to account for this perturbing acceleration on the orbital dynamics knowledge of the satellite attitude is necessary. Detailed radiation pressure models including surface properties of the satellite body and solar panels for reflected and scattered sunlight as well as reradiation of heat are available. Nevertheless, because the attitude of the satellites is not precisely known, orbit modeling problems may still occur, in particular during eclipsing periods. For the newer generation of GPS satellites (Block IIR) the system design allows for a predictable attitude motion. For the satellites of the older series elaborate models have been developed. At the CODE Analysis Center the problem is solved by estimating nine radiation pressure coefficients per satellite and per day. The time series of these parameters may be studied in order to identify satellites-specific problems. PRN 23 is a particularly interesting case. Its orbit was very difficult to model since its launch in 1990 due to hardware problems with the solar panel array. Early in 2002 the US DoD modified the attitude strategy for this satellite leading to a much better modelable orbit.

The GPS orbits provided by the IGS show a remarkable precision, today. The long standing discrepancy of ranges derived from SLR measurements with ranges derived from the orbits based on the microwave technique still needs explanation. SLR ranges seem to be about 6cm too short. A distinctive semi-annual signal in the SLR residuals for the two GPS satellites equipped with retroreflectors, in phase with the eclipsing periods, indicate that the cause of the problem almost certainly resides in the microwave technique. Further investigations are required to identify and eventually to eliminate this discrepancy.

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