# 14. Earth Orientation Modeling and Estimation

# 14.1 Motivation

By the term *Earth Orientation Parameter* (EOP) we understand a set of 5 parameters describing the orientation of the ITRF (*International Terrestrial Reference Frame*) in the ICRF (*International Celestial Reference Frame*) in conjunction with the conventional Precession-Nutation model. The parameters  $x_p$ ,  $y_p$  locate the *Celestial Ephemeris Pole* (CEP) in the terrestrial reference frame, whereas the position of the CEP in inertial space is defined by the IAU 1980 Theory of Nutation. The parameters  $d\psi$  and  $d\varepsilon$  reflect the difference between the actual position of the celestial pole and the position given by the conventional IAU model. Finally, the difference UT1–UTC (respectively UT1–TAI, TAI=International Atomic Time) gives access to the direction of the IERS reference meridian in the celestial reference frame (for details we refer to [*Castrique*, 1996]).

Station positions are conveniently represented in an Earth-fixed reference frame rotating in a welldefined way with the Earth. This reference frame is realized today by a catalogue of adopted geocentric coordinates and a velocity field of tracking stations derived from the analysis of data from various space techniques (VLBI, SLR, GPS, DORIS). The frame is designated as the *International Terrestrial Reference Frame (ITRF)* maintained by the International Earth Rotation Service (IERS). The latest realization is called ITRF97 with positions given at epoch  $t_0 = 1997.0$  [*Boucher et al.*, 1999].

The motion of the GPS satellites is best described in the *Inertial Celestial Reference Frame (ICRF)*, realized by a list of adopted equatorial coordinates (epoch J2000.0) of extra-galactic radio sources uniformly distributed over the sky.

In order to compute the difference vector between the observing station and the satellite both positions must be given in the same reference frame. Therefore, we need to know the current set of transformation parameters (EOP) between the two frames when analyzing GPS data. Moreover, when processing data from a global GPS tracking network, it is possible to estimate a subset of EOP parameters if the positions of some tracking sites are known in the ITRF.

# 14.2 Theory

The transformation between the Earth-fixed and the celestial coordinate system may be performed by means of equation

$$\tilde{\mathbf{r}} = \mathbf{X}^T \, \mathbf{Y}^T \, \mathbf{U} \, \mathbf{N} \, \mathbf{P} \, \mathbf{r} \tag{14.1}$$

where  $\tilde{\mathbf{r}}$  and  $\mathbf{r}$  denote the cartesian coordinates (column matrices) of a station in the terrestrial and inertial systems, respectively. The sequence of rotation matrices  $\mathbf{N} \mathbf{P}$  describes the transformation between the mean celestial system at epoch J2000.0 and a system defined by the true equator and equinox of date.  $\mathbf{P}$  and  $\mathbf{N}$  may be written as follows

$$\mathbf{P} = \mathbf{R}_3(-z) \mathbf{R}_2(\theta) \mathbf{R}_3(-\zeta)$$
(14.2a)

$$\mathbf{N} = \mathbf{R}_1(-\varepsilon_0 - \Delta \varepsilon) \, \mathbf{R}_3(-\Delta \psi) \, \mathbf{R}_1(\varepsilon_0) \approx$$

$$\approx \mathbf{R}_1(-\Delta\varepsilon) \mathbf{R}_2(\Delta\psi\sin\varepsilon_0) \mathbf{R}_3(-\Delta\psi\cos\varepsilon_0)$$
(14.2b)

where

$\mathbf{R}_i(lpha)$	•••	characterizes a rotation around axis <i>i</i> , about angle $\alpha$ ,
$z, heta,\zeta$		are the precession parameters,
$\varepsilon_0$		denotes the mean obliquity of the ecliptic, and
$\Delta \psi, \Delta \varepsilon$	•••	denote the nutation in longitude and obliquity, respectively.

 $\mathbf{U} = \mathbf{R}_3(\Theta_{GA})$  provides the transition to the rotating system where  $\Theta_{GA}$  is the Greenwich apparent sidereal time. Finally, the polar motion matrices

$$\mathbf{X} = \mathbf{R}_2(x_p) , \quad \mathbf{Y} = \mathbf{R}_1(y_p)$$
(14.3)

describe the separation between the third axis of the terrestrial system (the Conventional Terrestrial Pole = CTP) and the Celestial Ephemeris Pole (CEP).

To the accuracy level required *for the computation of the partial derivatives* of the GPS observable with respect to the parameters of interest, we may approximate the nutation matrix as a product of three infinitesimal rotations as shown in (14.2).

$$\mathbf{N} \approx \mathbf{R}_1(-\Delta \varepsilon) \ \mathbf{R}_2(\Delta \psi \sin \varepsilon_0) \ \mathbf{R}_3(-\Delta \psi \cos \varepsilon_0)$$

Introducing this result into Eqn. (14.1) we obtain the following simplified transformation equation:

$$\mathbf{r} = \mathbf{P}^{T}(t) \mathbf{R}_{1}(\Delta \varepsilon) \mathbf{R}_{2}(-\Delta \psi \sin \varepsilon_{0}) \mathbf{R}_{3}(-\Theta_{GM}) \mathbf{R}_{1}(y_{p}) \mathbf{R}_{2}(x_{p}) \tilde{\mathbf{r}}, \qquad (14.4)$$

where  $\Theta_{GM}$  stands for the Greenwich *mean* sidereal time. All Earth orientation parameters are contained in Eqn. (14.4). It is our goal now to derive expressions for the partial derivatives of the GPS observable with respect to these parameters. Neglecting refraction effects and leaving out range biases, we essentially observe the slant range  $\rho$  between the receiver position  $\mathbf{r}_k$  at observation time t and the GPS satellite position  $\mathbf{r}^i$  at time  $t - \rho/c$  where c stands for the velocity of light. The slant range may, e.g., be computed in the celestial reference frame:

$$\varrho = \sqrt{(\mathbf{r}^i - \mathbf{r}_k)^T (\mathbf{r}^i - \mathbf{r}_k)} .$$
(14.5)

Let p denote one of the Earth orientation parameters (e.g., a polar wobble component). From Eqn. (14.5), assuming that the partial derivative of the satellite position with respect to p is zero, we conclude

$$\frac{\partial \varrho}{\partial p} = -\mathbf{e}^T \; \frac{\partial \mathbf{r}_k}{\partial p} \tag{14.6}$$

where

$$\mathbf{e} = \frac{\mathbf{r}^i - \mathbf{r}_k}{\varrho} \tag{14.7}$$

is the unit vector (more precisely: cartesian components of the vector in the ICRF) pointing from the receiver to the satellite. For more information we refer to [*Kleusberg and Teunissen*, 1996].

Unfortunately, due to correlations with the orbital elements, a subset of the EOP is not directly accessible to the GPS (namely  $\Delta UT$  and the nutation parameters). This statement can easily be verified, e.g., for  $\Delta UT$  by comparing the partial derivative of  $\rho$  (see Eqn. (14.5)) with respect to the right ascension of the ascending node.

On the other hand, it is possible to solve for a drift in UT1 - UTC by adopting a model of the following type:

$$\Delta UT = \Delta UT_{t=t_0} + \frac{\mathrm{d}}{\mathrm{d}\,t} \left(\Delta UT\right)_{t=t_0} \cdot (t - t_0) \tag{14.8}$$

Due to this time dependence the length of day (LOD) may be estimated very well with the GPS.

# 14.3 Use of Earth Orientation Parameters in the *Bernese GPS Software*

#### 14.3.1 General Dataset Names

As mentioned previously, a set of EOP is required to perform the transformation from the celestial to the Earth-fixed system or vice versa. The programs PRETAB and BRDTAB (<u>Menu 3.2</u>) compute, e.g., a table of satellite positions in the inertial frame from the available orbit information (usually precise orbits in an Earth-fixed frame, e.g., ITRF94). Therefore, the EOP file corresponding to the orbits used has to be specified in <u>Panel 0.3.1</u> (Pole Information). If IGS precise orbits are used the corresponding EOP files (covering always one GPS week) are available at the global data centers. For example the "Crustal Dynamics Data Information System (CDDIS)"



stores, besides the precise ephemeris files, pole information stemming from different analysis centers. The Earth rotation parameters associated with the IGS ephemeris files are computed using a weighted average of all available center-specific ERP files. (The term Earth rotation parameters (ERP) is used for a 3-parameter subset of the EOP which comprises polar motion  $(x_p, y_p)$  and UT1.) The ERP-files stemming from CODE may be downloaded via anonymous ftp, too (see Chapter 7). Be aware that in contrast to the naming rules of the Bernese environment all these files contain pole information in the IERS format although their extension is ". ERP" (see also Chapter 24)! The pole files in IERS format have to be transformed to the Bernese pole format. This task is performed by program POLUPD (Menu 5.5.1) with options to be discussed in detail later.

If other than IGS precise orbits are used (and no specific ERP files are available), it is recommended to use the IERS C04 pole. C04 is a continuous, slightly low pass filtered series at daily intervals

and can be interpolated linearly. Oscillations in UT1 and in the length of day due to zonal tides for periods under 35 days, as well as the 14-day terms in  $x_p$  and  $y_p$  are present in full size in the series [*Castrique*, 1996].

The EOP series mentioned above may be obtained via anonymous FTP. Data files concerning Earth rotation (C04 series, IERS Bulletin B) and celestial frame are stored at the IERS Central Bureau and may be picked up as follows:



Quick-look EOP series (IERS Bulletin A, containing also predicted values) can be transferred from:



Information concerning updates of internet addresses and server names is always provided by the most recent IERS Annual Report. IERS pole files already transformed into the Bernese format may be downloaded via anonymous ftp, too. They are located in the directory ftp://ftp.unibe.ch/aiub/BSWUSER/GEN/ (files CO4\_\*.ERP, BULLET\_A.ERP, see also Chapter 7).

## 14.3.2 Update of Pole Information

In order to obtain the most accurate EOP information covering the span of your GPS campaign you are forced to update the file containing the Earth rotation parameter information, whenever more consistent (with the "precise orbit" data) and/or more recent values become available. The best way to perform this task is by means of the service programs POLUPD (Menu 5.5.1) and POLXTR (Menu 5.5.2).

POLUPD takes information from input files of (almost) any conventional format and converts the data to the Bernese pole format. Usually the extension ". IEP" indicates a file containing pole information in a foreign format, the extension ". ERP" denotes Bernese format. You have to keep this in mind whenever you leave input fields blank in Panel 5.5.1 and the menu system shows you a list of available input files.

Program POLUPD requires two files that have to be located in the X:/GEN directory. Their names have to be specified in Panel 0.3.1. The first file contains a table of all leap seconds during the last several years (see variable "LEAP SECOND" in Menu 0.3.1 and Chapter 24). You have to modify this file whenever a new leap second is announced by the IERS. The second file is the so-called Pole Offset Coefficient file. It contains values which have to be added to the EOP(IERS) time series in order to make them consistent with the current realization of the IERS terrestrial and celestial reference frames (see Chapter 24).

The example below illustrates the use of the subsequent Panel 5.5.1-1. The option NUTATION MODEL specifies the type of model you use to apply the nutation correction parameters. There are

three different entries. The HERRING model is a model derived from VLBI observations, OBSERVED specifies that nutation corrections given in columns DEPS and DPSI of the ERP file (to be created here) will be used. These corrections may stem from VLBI observations (e.g., in IERS Bulletin A) or from GPS observations computed by means of the Bernese software (research purpose). The recommended entry is NO. This choice prevents the use of nutation correction information and therefore the computations are solely based on the IAU 1980 nutation model.

```
5.5.1-1
                    POLE: UPDATE POLE FILE: GENERAL OPTIONS
Header information:
  TITLE
                                                                               <
                             - MU
> NO
                                         < (NO, OBSERVED, HERRING)
< (NO, RAY, GSFC, HERRING, NAIVE,</pre>
   NUTATION MODEL
   SUBDAILY POLE MODEL
                                               SPRINGER)
 Bulletin B as input:
   USE 1 OR 5 DAY VALUES
                                          < (1: one day values,
                              > 1
                                               5: five day values)
 Pole offsets:
                                          (NO or YES)
(NO or VES)
   ADD ERP OFFSETS (IERS)
                                > YES <
  USE ERP RATES
                                > NO <
                                             (NO or YES)
                                               5: five day values)
 Window:
  USE WINDOW
                                > NO <
                                              (YES.NO)
   FROM/TO
                         yyyy mm dd hh.hh
                                               yyyy mm dd hh.hh
                       > 1993 01 01 00.00 < > 1995 01 01 00.00 <
```

Program POLXTR (<u>Menu 5.5.2</u>) extracts ERP information from a list of consecutive pole files in the IERS format and allows you to specify an a priori pole file which may be used for comparison purposes. We refer to the corresponding help panel for details.

# 14.4 Estimation of Earth Orientation Parameters

Today, in view of the products made available through the IERS and the IGS it will in general *not* be necessary to solve for EOPs, but to use EOPs consistent with the orbits and coordinates used. For research type applications based on data stemming from a *global GPS network* it makes sense to set up EOPs. This section is important for *advanced users* of the *Bernese GPS Software* who wish to gain insight into *global applications of the GPS*. Let us start by mentioning a few basic facts:

- You will always estimate ERPs on top of an *a priori model*. These are characterized by a \*.ERP file. If you want to estimate the ERPs *from scratch* you have to use an a priori model with all parameters set to zero.
- The (empirical) model for the ERPs is set up in the program GPSEST, it may be modified in program ADDNEQ, where you have to be aware of the fact that you may *simplify* but not generalize the model in ADDNEQ.
- If you solve for ERPs you will probably process exactly *one session* of one day in GPSEST and produce longer arcs using ADDNEQ. You should be aware of the fact that for the ERP estimates you may divide the time interval of the session into a number of *subintervals* of equal length, and that within each subinterval each ERP (improvement on top of the a priori model) is represented by a polynomial.

- For diurnal and semi-diurnal variations of the ERPs, the model from [*Ray et al.*, 1994] is implemented in the *Bernese GPS Software* Version 4.2 and applied according to the file header.
- *UT*1-*UTC* estimates are special in one respect: due to the correlations with the orbital nodes mentioned above, it is *not* possible to solve for the *zero degree coefficient* of the polynomial pertaining to the first sub-interval of the session (if you introduce the orbital elements as unknowns in the same adjustment).

## 14.4.1 Options in GPSEST

**Panel 4.5–2.4** handles many special requests in connection with parameter estimation. You have to enable the parameter type EARTH ROTATION PARAMETERS. In this case the menu system gives you access to **Panel 4.5–2.4.4**, where you may first define the total number m of PARAMETER SETS (=total number of polynomials for each EOP). Specifying, e.g., m = 12 means that you use 2-hour bins to model Earth rotation (for 24-hour sessions).

1.5-2.4.4		PARAMETER	ESTIMATION:	EARTH F	ROTATION	PARAME	TERS	
Mode T(	el: DTAL NUMBER	OF PARAME	TER SETS	>	3 <			
Eart	th Kotation	(ERP) and	Nutation Pa	rameters fault a	s (NUT):	iamo		
X - Y - UT DI DI	-POLE -POLE I1-UTC ELTA EPSILOI ELTA PSI	> 2 < > 2 < > 2 < > 2 < > 2 < > 2 < > 0 < > 0 <	et (υ-4) αe	iauit a	priori s < < < <	1gma	(mas) (mas) (msec) (mas) (mas)	
CC	ONTINUITY BE ONSTRAIN DRI	ETWEEN SETS IFTS TO ZEI	S RO	> ERP > NO	< (NO, < (NO,	ERP, ERP,	NUT, BOTH) NUT, BOTH)	
# 01 BI	f Values pei ERNESE POLE	: Set Store FILE	ed in Files: > 3 <	IERS F	POLE FILE		> 3 <	

Subsequently you may select the number n of parameters for each of the five EOPs. The polynomial degree q is simply q = n - 1. n = 0 means that the corresponding parameters are *not* set up.

If you solve for polynomials of degree q > 0 you have to be aware of the fact that your estimates will in general *not* be continuous at the sub-interval boundaries. You may ask for continuity for the pole components  $x_p$ ,  $y_p$ , and for UT1 - UTC by specifying CONTINUITY BETWEEN SETS "ERP"; if you want to enforce continuity for the two nutation parameters you specify "NUT", and you specify "BOTH" if you want to have continuity for all parameters. Attention: if you ask for continuity in the case q = 0 (n = 1), this actually means that you model the EOPs by *one* parameter for the entire session! If you ask for continuity in the case q = 1 (n = 1), you actually model the EOPs as polygons (standard procedure at CODE). You might ask for continuity of the first derivative, an option that only makes sense for a polynomial degree q > 1. We recommend to use q = 1 and to ask for continuity. Information concerning the other options is available in the help panel corresponding to Panel 4.5–2.4].

It was mentioned that it is not possible to solve for all orbit parameters *and for UT1-UTC*. Therefore the menu system allows you to introduce a priori weights for each parameter in each sub-interval. You have to constrain UT as suggested in the panel below in order to avoid singularities.

4	. 5	-2.	4.4	ł	P	ARAI	METH	ER	ES	STIN	1AT I	ION:	Ε.	ARTI	HR	)T	ATI	CON PAR	AMETERS	(XP	, Y	'P, DT)	
			I	ROI	И						т	כ						SIG XP	SIG	YP		SIG DT	
	3	уу	mm	dd	hh	mm	SS			уу	mm	dd	hh	mm	SS			milli	arc sec	;	n	illi sec	
																		* ****	*.**	**		. ******	
	> 9	96	06	09	23	59	59	<	>	96	06	10	12	00	00	<	>		$\langle \rangle$		<>	.0000001	<
	> 9	96	06	10	12	00	00	<	>	96	06	11	00	00	00	<	>		$\langle \rangle$		<>		<
	> 9	96	06	11	00	00	00	<	>	96	06	11	12	00	01	<	>		$\langle \rangle$		<>		<

If you define the EOPs according to the above two panels, GPSEST will generate an output (in the general OUTPUT file) of the following type:

EARTI	H ROTA 	TION PARAME	TERS :			
CRD.	REQ.	(") (S) (DT	RMS ) RMS	(")/DAY (S)/DAY	RMS RMS	(")/DAY**2 (S)/DAY**2
X Y DT	1 1 1	0.0010080 0.0017345 0.0000000	0.0000900 0.0001013 0.0000000	-0.0007370 -0.0013315 -0.0002006	0.0001536 0.0001621 0.0000064	

## 14.4.2 Options in ADDNEQ

Program ADDNEQ (Menu 4.8.1) allows the user to combine normal equation systems generated by GPSEST. ADDNEQ allows you to re-consider some aspects of EOP estimation. The panel generated by ADDNEQ (Panel 4.8.1–2.3), see below) is similar to that generated by GPSEST, we only comment on some special features.

4.8.1-2.3	ADD	NORMAL	EQUTATION	SYSTEMS	: EARTH	ROTATION	PARAMETERS	5
Far	th Botation	n (FRD)	and Nutat:	ion Para	meters	(NUT) ·		
Lai	th hotatio	$\# \text{ of } p_2$	anu Nutat. remeters		nriori	sigma		
		# OI pa		1	u priori	fallering		
v		per se	L (U-4)	ISU PEC	luesi	TOTTOWING	requests	<i>(</i> )
X	-PULE	>	2 <	> 0.	<	> 0.	<	(mas)
Y	-POLE	>	2 <	> 0.	<	> 0.	<	(mas)
U	T1-UTC	>	2 <	> 0.000	001 <	> 0.	<	(msec)
D	ELTA EPSIL	DN >	2 <	> 0.000	)1 <	> 0.	<	(mas)
D	ELTA PSI	>	2 <	> 0.000	1 <	> 0.	<	(mas)
C	DNTINUITY I DNSTRAIN DI	BETWEEN RIFTS TO	SETS ZERO	> ERP > ERP	< <	(NO, (NO,	ERP, NUT, ERP, NUT,	BOTH) BOTH)
# # CI	PARAM. WI' PARAM. WI' DNSTRAIN D.	TH EQUAL TH EQUAL AILY RET	DRIFT (EH DRIFT (NU ROGRADE X-	RP) > 1 JT) > 1 -Y FREQU	< (1 < (1 VENCY TO	: CONTINUI : CONTINUI ZERO (0/1	TY BETW. I TY BETW. I ) > 0 <	)AYS) )AYS) ;
# o: B:	f Values po ERNESE POLI	er Set S <sup>.</sup> E FILE	tored in H > 3 <	Tiles:	IERS PO	LE FILE	> 3 <	

Let us assume that ADDNEQ obtains a series of \*.NEQ files pertaining to series of adjacent days. The resolution of the EOPs is given by the individual \*.NEQ files and will *formally not be altered by* ADDNEQ. You may ask for continuity of EOP estimations between subsequent days (SPECIAL OPTIONS = 2), or, you may ask that the EOPs are represented by *one polynomial of degree 1* over

the entire time interval covered by all the files (SPECIAL OPTIONS = 3). The program ADDNEQ2 allows to redefine (i.e. reduce) the resolution of the EOPs.

CONSTRAINING RETROGRADE TERMS is an option which is only of importance if you are interested in a sub-daily resolution of the EOPs. Please contact the Bernese team if you actually need this option!

There is one more important difference between the EOP estimation in GPSEST and ADDNEQ: whereas in GPSEST the corrections to the a priori pole are modeled as polynomials, the *absolute values and UT1R* (obtained from UT1 by removing the tidal variations with periods < 35 days) are represented as polynomials in ADDNEQ.

# 15. Initial Phase Ambiguities and Ambiguity Resolution

# 15.1 Motivation

The unknown number of cycles in the observation equations (9.17a) and (9.17b), the initial phase ambiguity parameters, have to be estimated in a first step as real-valued parameters. However, it is known, that these parameters actually have to be integer numbers. To *resolve* the ambiguities means (in our terminology) to assign the correct integer numbers to the real-valued estimates. Let us introduce the following notations:

- $\mathbf{x}_1$  ... is the column array containing all non-ambiguity parameters,
- $\mathbf{x}_1^0$  ... are the corresponding a priori values,
- $\mathbf{x}_2$  ... is the column array containing all ambiguity parameters,
- $\mathbf{x}_2^0$  ... are the corresponding a priori values, and
- $\bar{\mathbf{x}}_2^0$  ... are the corresponding known true (integer) numbers. Moreover
- $\Psi$  ... is the array containing the terms "computed" (model function),
- L ... is the array containing all measurements (terms "observed"),
- l, l' ... are the arrays of reduced measurements (terms "observed computed"),
- $\mathbf{w}, \mathbf{w}' \ldots$  are the arrays containing the residuals.

Let us inspect the effect of introducing the known (integer valued) ambiguities into the normal equation system. In the case of the float solution we may write the observation equations in matrix form as

$$(\mathbf{A}_1 \ \mathbf{A}_2) \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} - \underbrace{(\mathbf{L} - \Psi(\mathbf{x}_1^0, \mathbf{x}_2^0))}_{l} = \mathbf{w}$$
(15.1)

 $(A_1 \text{ and } A_2 \text{ are the parts of the first design matrix corresponding to the non-ambiguity resp. ambiguity parameters). The corresponding system of normal equations is$ 

$$\begin{pmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} \\ \mathbf{N}_{21} & \mathbf{N}_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1^T \mathbf{P} \mathbf{I} \\ \mathbf{A}_2^T \mathbf{P} \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{pmatrix}$$
(15.2)

(**P** is the weight matrix). Eliminating  $\mathbf{x}_2$  from Eqn. (15.2) we obtain

$$(\mathbf{N}_{11} - \mathbf{N}_{12}\mathbf{N}_{22}^{-1}\mathbf{N}_{21}) \cdot \mathbf{x}_1 = \mathbf{b}_1 - \mathbf{N}_{12}\mathbf{N}_{22}^{-1}\mathbf{b}_2 .$$
(15.3)

1

Assuming that the ambiguity parameters are known we may write

$$\mathbf{A}_{1}\mathbf{x}_{1} - \underbrace{(\mathbf{L} - \Psi(\mathbf{x}_{1}^{0}, \bar{\mathbf{x}}_{2}))}_{l} = \mathbf{w}', \qquad (15.4)$$

which gives

$$\mathbf{N}_{11}\mathbf{x}_1 = \mathbf{A}_1^T \mathbf{P} \mathbf{I}' = \mathbf{b}_1' \,. \tag{15.5}$$

We may write

$$- \mathbf{l}' = \Psi(\mathbf{x}_1^0, \bar{\mathbf{x}}_2) - \Psi(\mathbf{x}_1^0, \mathbf{x}_2^0) = \mathbf{A}_2 \cdot (\bar{\mathbf{x}}_2 - \mathbf{x}_2^0) = \mathbf{A}_2 \mathbf{d} \mathbf{x}_2$$
(15.6)

and therefore

$$\mathbf{N}_{11}\mathbf{x}_1 = \mathbf{A}_1^T \mathbf{P} \mathbf{l} - \mathbf{A}_1^T \mathbf{P} \mathbf{A}_2 \mathbf{d} \mathbf{x}_2 .$$
 (15.7)

This last equation shows how the normal equation system changes if the ambiguities have been resolved (fixed on their integer values). Fixing ambiguities considerably reduces the number of parameters and the solution will get much more stable. It should be pointed out, that usually the majority of unknown parameters actually are the ambiguities. How do the solutions improve if the ambiguities have been resolved? The answer depends strongly on the ratio between the number of unknown non-ambiguity parameters and the number of measurements which are used for the estimation of these parameters (the length of the observing sessions for static applications). Figure 15.1 shows the effect of ambiguity resolution if only the receiver coordinates and few troposphere parameters are estimated in a regional (European) network (for details see [*Mervart*, 1995]).

#### **RMS OF THE HELMERT TRANSFORMATION**



Figure 15.1: Rms of a 7-parameter Helmert transformation with respect to the "true" coordinate set.

In this case the main effect may be seen for session length up to 4 hours. However, the second important advantage of the ambiguity fixed solutions is the significantly reduced number of parameters which have to be stored in the memory. This saves RAM and speeds up processing considerably. If many parameters are estimated (orbits, Earth orientation parameters etc.) ambiguity resolution improves also the results of much longer sessions (3-days sessions used in CODE for IGS processing). Figure 15.2 shows the improvement in the estimated orbits (for details see [*Mervart et al.*, 1995]):



Figure 15.2: Orbit quality estimated from discontinuities at day boundaries (eclipsing and non-eclipsing satellites).

The ambiguity fixed solution is the official IGS CODE solution since June, 1995.

# 15.2 Theory

There are many methods how to resolve the ambiguities. Some of them are very sophisticated, some quite simple, but most of them consist of two steps:

- Step 1: The ambiguities are estimated as real numbers together with other parameters.
- Step 2: The integer values of the ambiguities are resolved using the results of Step 1 (the real-valued ambiguities and the variance-covariance matrix). Usually statistical tests are performed to resolve the ambiguities in a reliable way.

The *Bernese GPS Software* uses double-difference observations and therefore the double-difference ambiguities are estimated. Single difference (between receivers) ambiguities are then stored in the single difference header files. For each session and each baseline we have to select one single difference bias  $n_{Fk\ell}^j$  as reference and actually our estimated ambiguity parameters are the differences

$$n_{Fk\ell}^{ij} = n_{Fk\ell}^i - n_{Fk\ell}^j \,. \tag{15.8}$$

Usually, the ambiguity with the maximum number of observations is selected as reference. If there are N single difference ambiguities for one session and one baseline, there are at most N-1 linearly independent unknown ambiguity parameters. If there is an epoch when all the single difference phase measurements were initialized again, the session breaks up into two parts and for each part one reference ambiguity *must* be selected. In that case only N-2 ambiguity parameters may to

be estimated. The set of measurements corresponding to exactly one reference ambiguity is called an *observation cluster* (different from an ambiguity cluster, see below). In the following we will assume to have only one observation cluster.

Figure 15.3 shows the satellite visibility plot for a short (several minutes) session. For short sessions there is usually at least one satellite which was observed all the time. One of these satellites may be selected as reference satellite (and the corresponding ambiguity as reference ambiguity).



Figure 15.3: Satellite visibility plot for a short session and a short baseline.

For longer sessions the situation is different:

- No satellite is observed during the entire session,
- there are periods during which only few satellites were observed, for very long baselines there may be even periods during which only one or two satellites were observed.

Typically, for long baselines and sessions we obtain a satellite visibility plot as in Figure 15.4:



Figure 15.4: Satellite visibility plot for a long session and a long baseline.

In the case of Figure 15.4 the program selects (single difference) ambiguity  $A_1$  (maximum number of observations) as a reference. After the first ambiguity resolution step (real-valued ambiguities

are estimated) a detailed inspection usually shows that the (double difference) ambiguities  $A_2 - A_1, A_6 - A_1, A_7 - A_1, A_8 - A_1$  have large a posteriori rms errors. On the other hand, the parameters  $A_3 - A_1, A_4 - A_1, A_5 - A_1$  have small rms errors. This result is a consequence of the selection of the reference ambiguity. If  $A_2$  would have been selected as reference the parameters  $A_6 - A_2, A_7 - A_2, A_8 - A_2$  would have small a posteriori rms errors and the parameters  $A_1 - A_2, A_3 - A_2, A_4 - A_2, A_5 - A_2$  big ones. The following conclusions may be drawn:

- Depending on the selected reference certain differences between single difference ambiguities and the selected reference ambiguity are well established, other differences have large a posteriori rms errors.
- It is difficult to resolve *all* ambiguities if long sessions are processed because for each particular selection of a reference ambiguity some ambiguity parameters will have large a posteriori rms errors.

These considerations show that it is necessary to *optimize* the forming of (double) differences. Assuming that  $n_{Fk\ell}^j$  denotes our reference ambiguity, we are therefore resolving either the double-difference ambiguity parameter

$$n_{Fk\ell}^{ij} = n_{Fk\ell}^i - n_{Fk\ell}^j \tag{15.9}$$

directly or the difference between two of these terms

$$n_{Fk\ell}^{i_1i_2} = n_{Fk\ell}^{i_1j} - n_{Fk\ell}^{i_2j} \,, \tag{15.10}$$

which, as a matter of fact, is a double-difference ambiguity again. Every possible double-difference ambiguity is covered by one of the two equations and any double-difference ambiguity may be checked and possibly resolved. The resolved ambiguities are saved in the observation header files. We resolve the double-difference ambiguities, however, for book-keeping reasons, we store single difference ambiguities in the files. It does not make sense to say that a single difference ambiguity is resolved without specifying the reference ambiguity. Therefore, we introduce the term *ambiguity cluster*, which is the set of (single difference) ambiguities which are resolved relative to each other. In the single difference observation files the  $L_1$ ,  $L_2$  and  $L_5$  ambiguities are stored. (This actually is redundant because the  $L_5$  ambiguity is nothing else but the plain difference between the  $L_1$  and  $L_2$ ambiguities. The reason to store  $L_5$  ambiguities, too has to be seen in the fact that  $L_5$  ambiguities may sometimes be resolved a priori, see the ambiguity resolution strategies below.) Figure 15.5 shows the relevant part of a header file. Each ambiguity has its integer value (initialized to zero) and its cluster number. If two ambiguities refer to the same cluster number, this implies that they have been resolved relative to each other (see, e.g., ambiguities 1, 4, and 5 in Figure 15.5). Any double-difference ambiguity created from two such single difference ambiguities is then known. The ambiguities are set up by the programs SNGDIF and MAUPRP. Each satellite has at least one ambiguity.

AMB	SAT	EPOCH	WLF	L1-AMBIG.	CLUS	L2-AMBIG.	CLUS	L5-AMBIG.	CLUS
1	1	1	1/1	681360.	69	530932.	69	0.	1
2	1	2843	1/1	0.	2	0.	2	0.	2
3	1	2879	1/1	0.	3	0.	3	0.	3
4	31	1	1/1	633381.	69	493546.	69	0.	4
5	31	1108	1/1	-145987.	69	-113753.	69	0.	5
6	31	2758	1/1	0.	6	0.	6	0.	6
63	27	1	1/1	1551327.	69	1208828.	69	0.	81
67	16	1799	1/1	0.	67	0.	67	0.	86
68	16	2182	1/1	279589.	69	227907.	69	0.	87
69	2	1	1/1	1307750.	69	1019028.	69	0.	90
70	7	1	1/1	709357.	69	552748.	69	0.	93
71	5	1800	1/1	1334718.	69	1040042.	69	0.	97
72	6	1	1/1	1518492.	69	1183242.	69	0.	100

Figure 15.5: Ambiguities stored in single difference phase header file.

# 15.3 Ambiguity Resolution Algorithms

There are four ambiguity resolution strategies implemented in the *Bernese GPS Software* Version 4.2: ROUND, SIGMA, SEARCH and QIF which may be selected in menu 4.5–1 (strategies ELIMIN and N0 do not try to resolve any ambiguities). Different algorithms use different combinations of the  $L_1$  and  $L_2$  observations. This aspect will be discussed in Section 15.4. Let us mention, that for the QIF strategy both  $L_1$  and  $L_2$  observations are required, the SEARCH strategy gives the best results with observations on both frequencies, too, and that the remaining strategies ROUND and SIGMA work actually with one carrier (or one linear combination) only. It is possible to use 2 linear combinations in one run, but the ambiguity resolution is performed independently on both carriers.

# 15.3.1 NO Algorithm

If you specify "YES" for option INTRODUCE WIDELANE or INTRODUCE L1 AND L2 in Panel 4.5–1, the ambiguities resolved in previous runs will be introduced as known values. All other ambiguities (or all ambiguities, if you specify "NO" for option INTRODUCE) will be introduced as unknown parameters and estimated. These ambiguities may or may not be pre-eliminated according to options in Panel 4.5–2.4.8.

## 15.3.2 ELIMIN Algorithm

The only difference between NO and ELIMIN is that if you select ELIMIN strategy the ambiguities will be pre-eliminated before the normal equation matrix inversion. It is just the same as if you select NO strategy and "BI" option for ambiguity parameters in Panel 4.5–2.4.8.

#### 15.3.3 ROUND Algorithm

This is the simplest ambiguity resolution strategy which only rounds the real-valued estimates to the nearest integers without using any variance-covariance information. Actually there is usually no need to use this strategy. The SIGMA strategy can do exactly the same work, if necessary. We do not recommend to use the strategy ROUND on baselines longer than a few kilometers. It is not a safe strategy.

#### 15.3.4 SIGMA Algorithm

Let  $x_i$ ,  $x_j$  be two (double-difference) ambiguity parameters (relative to the same reference ambiguity). For each parameter  $x_i$  we compute the a posteriori rms error in the initial least-squares adjustment:

$$m_i = \sigma_0 \sqrt{Q_{ii}} \,, \tag{15.11}$$

where  $Q_{ii}$  is the corresponding element of the cofactor matrix. For the difference  $x_i - x_j$  the a posteriori rms error is

$$m_{ij} = \sigma_0 \sqrt{Q_{ii} - 2 \cdot Q_{ij} + Q_{jj}} \,. \tag{15.12}$$

The rms errors  $m_i$  and  $m_{ij}$  of every possible double-difference ambiguity (see Eqn. (15.9) and (15.10)) are first sorted in ascending order of their rms errors. Within one iteration step the  $N_{max}$  best determined ambiguities (or differences between ambiguities) are then resolved (rounded to nearest integers), provided

- the corresponding a posteriori rms error  $m_i, m_{ij}$  is compatible with  $\sigma_0$  ( $m_i \leq \sigma_{max}$ , or  $m_{ij} \leq \sigma_{max}$ ), and
- that within the confidence interval  $(x_i \xi m_i, x_i + \xi m_i)$  or  $(x_{ij} \xi m_{ij}, x_{ij} + \xi m_{ij})$  there is exactly one integer number.

 $N_{max}$ ,  $\sigma_{max}$  and  $\xi$  are input parameters of the program GPSEST (see Panel 4.5–1.1). In the next iteration step the integer values are introduced for the resolved ambiguities and for the resolved differences between ambiguities (see Eqn. (15.7)). The iteration process is terminated, if:

- 1) all ambiguities have been resolved, or if
- 2) in the last step no ambiguity could be resolved based on the above criteria.

The iteration process described above may be applied to every linear combination. It may be used in the baseline mode, in the session mode, or even if several sessions are treated in the same program run. We recommend to use this strategy in two cases:

- 1) Only single-frequency measurements are processed, but the session is long (several hours). The baselines should not be to long (less than 20 km).
- 2) High quality code measurements are available on both frequencies. In this case it is possible to use the Melbourne-Wübbena linear combination and the corresponding strategy (see Section 15.4). The baselines may be very long (up to several thousand kilometers). The sessions have to be long too (several hours).

#### 15.3.5 SEARCH Algorithm

The SEARCH strategy is related to the so-called FARA (Fast Ambiguity Resolution Approach) algorithm (see [*Frei and Beutler*, 1990], [*Frei*, 1991]). The following information from the initial least-squares adjustment is used:

- $\mathbf{x} = (x_1, \dots, x_u)^T$ , the part of the solution vector consisting of all real-valued (double-difference) ambiguities, where u is the number of double-difference ambiguities,
- ${\bf Q},$  the corresponding cofactor matrix, and
- $\sigma_0^2$ , the a posteriori variance factor.

From the a posteriori variance factor and the corresponding cofactor matrix the standard deviation  $m_i$  for the ambiguity parameter  $x_i$  or the standard deviation  $m_{ij}$  for the difference  $x_{ij}$  between two ambiguity parameters  $x_i$ ,  $x_j$  may be computed:

$$m_i = \sigma_0 \sqrt{Q_{ii}}, \quad m_{ij} = \sigma_0 \sqrt{Q_{ii} - 2 \cdot Q_{ij} + Q_{jj}}.$$
 (15.13)

Choosing a confidence level  $\alpha$  and using Student's *t*-distribution we compute the upper and lower range-width  $\xi$  for the integer valued alternative parameter  $x_{Ai}$  or for the difference  $x_{Aij}$  between two such parameters. Thus

$$x_i - \xi \cdot m_i \le x_{Ai} \le x_i + \xi \cdot m_i, \quad i = 1, 2, \dots, u$$
 (15.14)

$$x_{ij} - \xi \cdot m_{ij} \le x_{Aij} \le x_{ij} + \xi \cdot m_{ij}, \quad i, j = 1, 2, \dots, u, \quad i \neq j.$$
(15.15)

All possible combinations of integer values which meet the conditions (15.14) and (15.15) are used to form alternative ambiguity vectors

$$\mathbf{x}_{Ah}, \ h=1,\ldots,N$$

to the initial ambiguity estimate  $\mathbf{x}$ . These alternatives are generated in forming all possible combinations of vector components using the integer values within corresponding confidence ranges. Each of these alternative vectors is introduced into a subsequent adjustment. The integer ambiguities are treated in these adjustments as known quantities. The resulting standard deviations

$$\sigma_h, h = 1, \ldots, N$$

are indicators for the success of the process: the integer vector  $\mathbf{x}_h$  yielding the smallest standard deviation is selected as the final solution, unless

- 1) its standard deviation is not compatible with the standard deviation  $\sigma_0$  of the ambiguity-free solution (the fraction  $\sigma_h/\sigma_0$  is too high), or
- 2) there is another vector  $\mathbf{x}_q$  yielding an almost identical standard deviation (fraction  $\sigma_q/\sigma_h \approx 1$ ).

The maximum allowed fraction  $(\sigma_h/\sigma_0)_{max}$  and the minimum discrimination fraction  $(\sigma_q/\sigma_h)_{min}$  are input options in Panel 4.5–1.3. In order to reduce the computation time and decrease the number of alternative vectors one more condition is introduced if both frequencies  $(L_1 \text{ and } L_2)$  are processed. Using the geometry-free linear combination (see Chapter 9) we may write

$$L_{4k\ell}^{ij} + I_{k\ell}^{ij} \left( 1 - \frac{f_1^2}{f_2^2} \right) = \lambda_1 x_{1k\ell}^{ij} - \lambda_2 x_{2k\ell}^{ij} , \qquad (15.16)$$

if the real-valued ambiguities  $x_{1k\ell}^{ij}$ ,  $x_{2k\ell}^{ij}$  are taken into account. Instead of these real-valued ambiguities we may use the alternative (integer) values  $x_{A1k\ell}^{ij}$ ,  $x_{A2k\ell}^{ij}$  to compute the right-hand site of Eqn. (15.16):

$$\lambda_1 x_{A1k\ell}^{ij} - \lambda_2 x_{A2k\ell}^{ij}$$

The difference

$$| (\lambda_1 x_{1k\ell}^{ij} - \lambda_2 x_{2k\ell}^{ij}) - (\lambda_1 x_{A1k\ell}^{ij} - \lambda_2 x_{A2k\ell}^{ij}) |$$

is actually the difference between the ionosphere bias which was estimated during the initial ambiguity-free solution and the ionosphere bias which would be the result of the alternative ambiguity-fixed solution. The difference has to be very small (see option SEARCH WIDTH FOR GEOMETRY-FREE LC in Panel 4.5–1.3).

It is almost mandatory to use the SEARCH strategy in rapid static mode. If both frequencies are available ( $L_1$  and  $L_2$  measurements are processed) usually several minutes of data are sufficient to resolve the ambiguities and achieve an accuracy of about a centimeter. If only one frequency is processed the observation interval has to be longer (usually about 30 minutes of data are sufficient). In rapid static mode usually only short (up to several kilometers) baselines are processed.

The disadvantage of our SEARCH strategy has to be seen in the fact, that either all the ambiguities or none are resolved. This may cause problems if long sessions are processed and/or very long baselines are involved (see Figure 15.4).

#### 15.3.6 QIF (Quasi-Ionosphere-Free) Algorithm

We neglect the troposphere bias in Eqns. (9.17a) and (9.17b) and do not explicitly write the receiver and satellite indices  $k, \ell, i, j$ . Then the simplified form of the double difference observation equations reads as

$$L_1 = \varrho - I + \lambda_1 n_1 \tag{15.17}$$

$$L_2 = \rho - \frac{f_1^2}{f_2^2} \cdot I + \lambda_2 n_2$$
 (15.18)

The corresponding equation for the ionosphere-free linear combination may thus be written as

$$L_3 = \rho + B_3 = \rho + \frac{c}{f_1^2 - f_2^2} (f_1 n_1 - f_2 n_2) .$$
 (15.19)

The initial least-squares adjustment using both frequencies  $L_1$  and  $L_2$  gives real-valued ambiguity estimates  $b_1$  and  $b_2$  and we may compute the corresponding ionosphere-free bias  $\tilde{B}_3$  as

$$\tilde{B}_3 = \frac{c}{f_1^2 - f_2^2} \left( f_1 \ b_1 - f_2 \ b_2 \right) \ . \tag{15.20}$$

This bias may be expressed in narrow-lane cycles (one cycle corresponding to a wavelength of  $\lambda_3 = c/(f_1 + f_2) \approx 11$  cm, see Chapter 9):

$$\tilde{b}_{3} = \frac{\tilde{B}_{3}}{\lambda_{3}} = \tilde{B}_{3} \cdot \frac{f_{1} + f_{2}}{c} = \frac{f_{1}}{f_{1} - f_{2}} b_{1} - \frac{f_{2}}{f_{1} - f_{2}} b_{2}$$
$$= \beta_{1} b_{1} + \beta_{2} b_{2} .$$
(15.21)

Denoting the correct (resolved) integer ambiguity values by  $n_{1i}$  and  $n_{2j}$  (*i* and *j* are not the satellite indices) and introducing the associated  $L_3$ -bias

$$b_{3ij} = \beta_1 \ n_{i1} + \beta_2 \ n_{2j} \tag{15.22}$$

we may use the difference between the real-valued and integer  $L_3$ -bias

$$d_{3ij} = |\tilde{b}_3 - b_{3ij}| \tag{15.23}$$

as a criterion for the selection of the "best" pair of integers  $n_{1i}$ ,  $n_{2j}$ . However, many pairs  $n_{1i}$ ,  $n_{2j}$  give differences  $d_{ij}$  of the same (small) order of magnitude. These pairs lie on a narrow band in the  $(n_1, n_2)$  space. The equation for the center line of this band is

$$\beta_1 n_{i1} + \beta_2 n_{2j} = \tilde{b}_3 . \tag{15.24}$$

The band-width is essentially given by the rms of the bias  $\tilde{b}_3$ . A unique solution only results if it is possible to limit the search range. The principle is shown in Figure 15.6.



**Figure 15.6:** Search ranges in  $(n_1, n_2)$  space.

The solid line corresponding to Eqn. (15.24) goes through the real valued estimate  $(b_1, b_2)$  (shown as o in figure) as well as through the point  $(n_{1,i}, n_{2,j})$  which is accepted as "true" solution. This line represents an ionosphere-free combination (constant ionosphere-free bias). The second solid in Figure 15.6 represents the constant wide-lane ambiguity (accepted as "true" value) and goes through the point  $(n_{1,i}, n_{2,j})$ , too. The dashed rectangle represents a search range in  $(n_1, n_2)$  space and the dashed trapezoid represents the search range in  $(n_1, n_5)$  space — Eqn. (15.29).

#### 15.3.6.1 The Role of the lonosphere

For baselines longer than about 10 km processing of the two frequencies  $L_1$  and  $L_2$  separately does *not* give sufficiently good initial real valued estimates  $b_1$  and  $b_2$  due to the influence of the

ionospheric refraction. Two types of models to reduce the ionospheric biases are considered (see also Chapter 13):

1) Satellite and Epoch Specific lonosphere Estimation: one ionospheric correction  $I_k^i(t_j)$  for satellite *i*, receiver *k* and epoch  $(t_j)$  is estimated. Estimating these parameters without any a priori constraints would be equivalent to processing the ionosphere-free linear combination. If we want to resolve the integer ambiguities it is necessary to constrain these parameters to within a few decimeters. This constraining may be achieved by introducing an artificial observation

$$I_k^i(t_j) - I_{k,apr}^i(t_j) = 0 (15.25)$$

for each epoch with a non-zero a priori weight. The actual values  $I_{k,apr}^{i}(t_j)$  may stem from an ionosphere model, in many cases (baselines up to 500 km) even  $I_{k,apr}^{i}(t_j) = 0$  may be sufficient. It is of course necessary to pre-eliminate all epoch-specific ionosphere parameters  $I_{k,apr}^{i}(t_j)$ ,  $i = 1, 2, ..., n_s$  ( $n_s$  is the number of satellites per epoch) after having processed epoch  $t_j$ , because a "terrible" number of parameters would have to be handled in the normal equation system after  $n_e$  epochs.

2) Deterministic Model: single-layer models developing the electron content in a layer of infinitesimal thickness in a height of about 350 km above the surface of the Earth into a series of harmonical coefficients in latitude and hour angle of the Sun. Such a model should be used if long baselines (500 km - 2000 km) are processed.

A combination of the two types of models may be used.

#### 15.3.6.2 Implementation of the QIF Strategy

Let us denote by  $b_{1i}$ ,  $b_{1i_1}$ ,  $b_{1i_2}$  the (real-valued) double difference  $L_1$  ambiguities. Similarly by  $b_{2j}$ ,  $b_{2j_1}$  and  $b_{2j_2}$  the corresponding  $L_2$  ambiguities. Now, we check whether the pair

$$b_{1i}$$
,  $b_{2j}$ 

or the pair

$$b_{1i_1} - b_{1i_2}$$
,  $b_{2j_1} - b_{2j_2}$ ,

which, as a matter of fact, is a pair of double-difference ambiguities again, meets the requirements to be close to integers and may be accepted as the correct pair of integer ambiguities. Let us explain the procedure in more detail. We compute the rms error for each  $L_3$  ambiguity bias  $\tilde{b}_3$  associated with a pair  $b_{1i}$ ,  $b_{2j}$  or with a pair of differences  $b_{1i_1} - b_{1i_2}$ ,  $b_{2j_1} - b_{2j_2}$ :

$$\sigma = \sigma_0 \cdot \sqrt{\beta_1^2 Q_{11} + 2 \beta_1 \beta_2 Q_{12} + \beta_2^2 Q_{22}}, \qquad (15.26)$$

where

$$Q_{11} = Q(b_{1i}, b_{1i}), \quad Q_{12} = Q(b_{1i}, b_{2j}), \quad Q_{22} = Q(b_{2j}, b_{2j})$$
 (15.27)

in the case of pair  $b_{1i}, b_{2j}$  (Q(...) is an element of the cofactor matrix) or

$$Q_{11} = Q(b_{1i_1}, b_{1i_2}) - 2 Q(b_{1i_1}, b_{1i_2}) + Q(b_{1i_1}, b_{1i_2})$$

$$Q_{12} = Q(b_{1i_1}, b_{2j_1}) - Q(b_{1i_1}, b_{2j_2}) - Q(b_{1i_2}, b_{2j_1}) + Q(b_{1i_2}, b_{2j_2})$$

$$Q_{22} = Q(b_{2j_1}, b_{2j_1}) - 2 Q(b_{2j_1}, b_{2j_2}) + Q(b_{2j_2}, b_{2j_2})$$
(15.28)

in the case of pair of differences  $b_{1i_1} - b_{1i_2}$ ,  $b_{2j_1} - b_{2j_2}$ . We sort the ambiguity pairs in ascending order of their  $\sigma$ -values. For the ambiguity pair (or pair of the differences) with the smallest  $\sigma$  (if this  $\sigma$  is lower than the specified  $\sigma_{\max}$ ) we define the search ranges

$$\tilde{n}_{1} = \operatorname{nint}(\mathbf{b}_{1}) \pm \mathbf{i}, \quad \mathbf{i} = 0; 1; \dots; \mathbf{i}_{\max} 
\tilde{n}_{5} = \operatorname{nint}(\mathbf{b}_{1} - \mathbf{b}_{2}) \pm \mathbf{k}, \quad \mathbf{k} = 0; 1; \dots; \mathbf{k}_{\max} 
\tilde{n}_{2} = \tilde{n}_{1} - \tilde{n}_{5}$$
(15.29)

and for each pair  $\tilde{n}_1, \tilde{n}_2$  of integers within the search range we compute the test value (15.23)

$$d_3 = \left|\beta_1 \left(b_1 - \tilde{n}_1\right) + \beta_2 \left(b_2 - \tilde{n}_2\right)\right|.$$
(15.30)

The pair associated with the smallest value  $d_3$  is accepted as a solution, unless

$$d_3 \ge d_{\max} , \qquad (15.31)$$

where  $d_{\text{max}}$  is a user-defined maximum value. If no ambiguity set passed the test we proceed to the next pair of ambiguities associated with the second smallest  $\sigma$ . After having accepted one pair the entire least-squares adjustment and the procedure described above are repeated. The ambiguities are thus resolved iteratively. All or only a subset of ambiguity pairs may be resolved in the iteration process.

# 15.4 Ambiguity Resolution Strategies

Let us now give comments and recommendations concerning the selection of the appropriate ambiguity resolution strategy. All algorithms described in the previous section may be used either in the baseline mode or in the session mode. However, for the sake of efficiency, we recommend to resolve the ambiguities in the baseline mode (processing each baseline separately) and then introduce the resolved ambiguities as known quantities into the subsequent session processing. There are several aspects determining the optimal ambiguity resolution strategy. Before selecting the strategy the following questions should be answered:

- 1) Are phase measurements available on both carriers? If yes, are there high quality code measurements on both carriers, too? (This is often not true under the A/S regime.)
- 2) How long is the baseline?
- **3**) How long is the session?

If only single frequency data are processed, there are not many possibilities how to resolve the ambiguities. Either the SEARCH algorithm (for short sessions) or the SIGMA algorithm (for long sessions) must be used. Only short (several kilometers) baselines should be processed.

If both frequencies are available, we have the following options:

1) For very short baselines (up to several kilometers) the ambiguities may be resolved independently on  $L_1$  and  $L_2$  using the SIGMA algorithm (it is possible to do that in one program run, if both carriers are processed). This case is similar to processing single frequency data.

- 2) For longer baselines (up to approximately 2000 km) it is possible to process both carriers together and to try to resolve both ( $L_1$  and  $L_2$ ) ambiguities in the same run. The recommended strategies are QIF for long sessions and baselines of almost arbitrary length and SEARCH for short sessions and short baselines.
- 3) Use of the so-called wide-laning technique. In this case each baseline is processed twice. First the  $L_5$  or Melbourne-Wübbena linear combination is processed, the wide-lane  $(L_5)$  ambiguities are resolved and stored in the observation header file. In the subsequent run the  $L_3$  linear combination is processed, the wide-lane ambiguities are introduced as known, and the narrow-lane  $(L_1)$  ambiguities are resolved.

It was stated above that in the first step of the third strategy (wide-laning technique) either the  $L_5$  or the Melbourne-Wübbena linear combination is processed. Using  $L_5$  (phase observations only) gets satisfactory results up to baseline lengths of about 100 km (or longer if a good a priori ionosphere model is used). In this case the processing steps are:

- Ambiguity-free  $L_3$  solution saving the coordinate results to get good a priori coordinates for the next step.
- Wide-lane  $(L_5)$  ambiguity resolution fixing *all* coordinates on the values obtained in the previous step.
- $L_3$  solution introducing the resolved  $L_5$  ambiguities and solving for the *narrow-lane* ambiguities. In this third step the estimation of station-specific troposphere parameters is highly recommended (see Chapter 12 because the geometrical term  $\rho$  (including the tropospheric refraction) has to be accurately known considering the small wavelength. Note that the estimation of troposphere parameters is always recommended for long baselines when doing narrow-lane ambiguity resolution or when using the QIF strategy.

The approach given above (using phase observations only) does not seem to have advantages over the QIF strategy.

Resolving wide-lane ambiguities using the Melbourne-Wübbena linear combination is very reliable and almost baseline-length independent. However, the quality of the code measurement is crucial.

The recommended strategies are summarized in Table 15.1.

	Baseline length							
	short medium long							
	< 10 km	10–100 km	100–2000 km					
short	SEARCH	SEARCH	-					
session								
long	QIF or	QIF	QIF or					
session	SIGMA $L_1 \& L_2$		SIGMA Wuebb.					

 Table 15.1: Ambiguity resolution strategies.

# 15.5 Resolving GLONASS-Related Ambiguities

In case of processing GLONASS data, ambiguity resolution is much more demanding than in case of GPS. The reason for the difficulty comes from different GLONASS carrier frequencies, which are specific to the satellites' channel numbers. As a consequence of this, one is forced to take into consideration so-called single-difference bias terms, which remain relevant on the double-difference level. The interested reader is referred to [*Habrich*, 1999], where the GLONASS-related ambiguity resolution is discussed in detail.

#### **Restriction**

If you process GLONASS-only or GPS/GLONASS-mixed data using the *Bernese GPS Software* Version 4.2, only the following ambiguity resolution strategies should be used: "NO", "ELIMIN", "SIGMA". In other words, "SIGMA" is *the* ambiguity resolution strategy as soon as GLONASS tracking data is considered, since strategies such as "NO" and "ELIMIN" do not actually "resolve" ambiguities.

# 16. Processing Undifferenced Data

Although the *Bernese GPS Software* is primarily focused on processing double-differenced GPS observations, it is now also capable of processing undifferenced GPS observations. The two most important reasons for this enhancement of our software are that processing undifferenced data gives access to the satellite and receiver clocks and it allows for a so-called precise point positioning.

In principle the undifferenced or zero-differenced (ZD) processing can also be used for all "classical" applications of the *Bernese GPS Software* like, e.g., coordinate estimation, saving of normal equations, etc. We, however, advise you to use the double-difference (DD) approach for this because that is the primary estimation procedure of the *Bernese GPS Software*. Furthermore, the undifferenced processing currently does not support ambiguity resolution.

There are no fundamental differences between the results obtained using undifferenced or doubledifferenced observations. There are, however, some practical differences. Compared to the DD approach the ZD approach has as disadvantage that the data cleaning is more difficult and that the data cleaning algorithm implemented in the *Bernese GPS Software* is very dependent on the quality of the code observations and it requires dual frequency code and phase observations. Furthermore, the number of parameters which have to be estimated is relatively large, due to the additional clock parameters. These clock parameters may be pre-eliminated if the user is not interested in them. The DD approach, as implemented in the *Bernese GPS Software*, has as disadvantage that less observations are used compared to the ZD approach. This effect is insignificant for relatively dense regional networks as mostly encountered today within the GPS technique. However, in large but sparse networks the reduction in the number of observations may be significant.

Additionally, the implementation of the undifferenced processing has allowed us to implement the processing of Satellite Laser Ranging (SLR) data. Although we do not envision this to be something that will be widely used, we will briefly describe how to process SLR observations using the *Bernese GPS Software*.

The ZD approach using combined GPS/GLONASS stations does not work in Version 4.2 of *Bernese GPS Software*. There is an offset between the GPS and GLONASS time frame which may differ up to several hundreds of nanoseconds. This is still not handled in the software up to now.

# 16.1 Cleaning of Undifferenced Data

The main problem when processing undifferenced GPS data is to reliably and automatically detect outliers and cycle slips in the observations. The data cleaning of differenced GPS data is much easier because many common error sources may be removed by forming the differences, in particular station and satellite clocks. However, several programs have been developed which seem to be reasonably successful in cleaning undifferenced GPS data. For the development of our automatic editing algorithm for undifferenced GPS data we have taken an approach similar to that used by the TurboEdit program [*Blewitt*, 1990].

Our algorithm, like the TurboEdit program, requires the use of dual frequency code and phase observations. The major problem with this approach resides in the fact that it depends heavily on the quality of the code observations. For TurboEdit the noise of the code observations is assumed to be below 0.5 wide-lane cycles, i.e., 43 cm. This requirement is easily fulfilled if anti-spoofing (AS) is not active. For most state-of-the-art geodetic receivers the noise of the code observations under those conditions is at the 20 cm level. However, with AS active, the noise is significantly larger because only C/A-code is available on the  $L_1$  frequency and the code on the  $L_2$  frequency is reconstructed using either so-called cross-correlation or W-code tracking techniques [Ashjaee and Lorenz, 1992].

The input to our program RNXSMT, which stands for RINEX SMooThing, is a single RINEX file. The output is a RINEX file again, hopefully free from outliers and cycle slips. A summary of the actions taken by the program is contained in the output. Each RINEX file is processed satellite by satellite. The observations of each satellite are processed in four steps:

- (1) Screening of the Melbourne-Wübbena linear combination, Eqn. (9.28), for outliers and cycle slips.
- (2) In case cycle slips are detected in the first step, the geometry-free linear combination, Eqn. (9.24), is checked in order to determine the size of the cycle slip on both frequencies. This information can be used to connect the observations before and after the cycle slip. Currently only the smoothed code data pieces (arcs) may be "connected". The phase observations are never connected over a cycle slip; a new ambiguity is set up at the epoch of the detected cycle slip.
- (3) Screening of the difference between the code and phase ionosphere-free linear combinations, Eqns. (9.19 and 9.20); i.e.,  $L_3 P_3$ . This screening is performed to remove bad observations which were accepted in the Melbourne-Wübbena screening.
- (4) Smooth the code observations using the code and carrier phase data of the clean observation arcs.

Below, each of these steps is discussed in detail. Additional information may also be found in [Springer, 1999].

## 16.1.1 Data Screening Based on Melbourne-Wübbena Linear Combination

The Melbourne-Wübbena combination eliminates the effects of the ionosphere, geometry, clocks and the troposphere (see Eqn. (9.28)). Apart from the wide-lane ambiguity the remaining signal should be pure noise, with an rms error of approximately 0.7 times the rms of the code observations on the  $L_1$  frequency. If the noise of the Melbourne-Wübbena combination has an rms error below 0.5 wide-lane cycles (43 cm) it is almost trivial to detect all cycle slips and outliers. Only very few epochs are needed to estimate the wide-lane ambiguity, jumps and outliers can easily be detected. Of course, only the difference between the cycle slips on the two frequencies is detected ( $n_5 = n_1 - n_2$ ),



Figure 16.1: Noise of the Melbourne-Wübbena combination under different AS conditions. Data of one station (Wettzell, Germany) collected during two days in 1997 is shown.

see Eqn. (9.28). Note that in the very unlikely case where the integer number of cycle slips on the two frequencies are identical (i.e.,  $n_1 = n_2$ ) no cycle slip will be detected ( $n_5 = 0$ ).

However, under AS the noise of the Melbourne-Wübbena combination for most geodetic receivers exceeds the rms of 0.5 wide-lane cycles. Figure 16.1 shows the effect of AS on the Melbourne-Wübbena combination. It shows a single pass of a single satellite (PRN 18) for the same station on two different days. On the first day, day 40, 1998, AS was not activated. On the second day, day 60, 1998, AS was active. The different noise level on the two days is quite obvious. The beginning and the end of the observation arc illustrate that for low elevations a much more pronounced increase of the noise level is observed for "AS on" as compared to "AS off". For low elevations it will thus be more difficult to detect small outliers and cycle slips. It will be impossible to detect outliers of one or two wide-lane cycles (86–172 cm). It is also interesting to note the shift in time of the observation arc of this satellite. On day 40 the satellite was first tracked at approximately 1:30 hours, whereas on day 60 the first data of the satellite were observed at approximately 0:10 hours. This is caused by the 4 minute shift per day of the satellite—station geometry.

The only way to improve the reliability of screening data from one station is to generate as long arcs as possible. So, instead of using a running average, as it is done in the TurboEdit algorithm, we make the attempt to use all observations within one satellite pass. An arc is defined by specifying a minimum number of observations and a maximum time for data gaps. Typical values are a minimum of 10 data points per arc and a maximum of 3 minutes without observations before starting a new arc.

After defining the arcs, the rms error of the observations in the arc is computed. If the rms exceeds a user-specified maximum the observation arc is screened for cycle slips. In the cycle slip detection the observation arc is split-up into two equally long parts. It is assumed that the part with the larger rms contains the cycle slip(s). Both parts are edited for outliers, using as outlier level four times the rms ( $4\sigma$ ) of the observations in the arc with a maximum rms value specified by the user. In this step outliers are only temporarily removed. The difference between the two (clean) parts is estimated and the whole arc is connected using the estimated cycle slip. All points that were considered outliers during the cycle-slip detection are included again. The rms is recomputed to check whether there are more cycle slips in this observation arc.

After the detection of all cycle slips, the observations are screened for outliers. Outliers are removed until the rms of the observation arc is below the specified maximum. Outliers are those points which exceed a value of  $4\sigma$ . The specified maximum rms is typically 0.4–0.6 wide-lane cycles (34–52 cm). If an outlier is detected, all four observation types (code and phase on two frequencies) are rejected.

## 16.1.2 Data Screening Based on Geometry-Free Linear Combination

Only those observation arcs in which cycle slips have been detected are screened using the geometry-free combination of the phase observations (see Eqn. (9.24)). At this stage the size of the wide-lane cycle slip  $(n_1 - n_2)$  is known. The geometry-free linear combination  $(L_4)$  allows us to compute the size of the  $n_1$  and  $n_2$  cycle slips because it gives us:  $L_4 = L_1 - L_2$ . To determine the size of the cycle slip on the  $L_4$  linear combination two linear polynomials are fitted through n points, "n" defined by the user, before and after the cycle slip. The difference between the two polynomials at the time of the cycle slip is computed. If the fractional part of the difference is smaller than a user specified limit the  $n_4$  cycle slip is accepted and the  $n_1$  and  $n_2$  cycle slips are computed. Typically a value of n=10 is used and a difference smaller than 10 mm. This procedure is only executed to be able to connect the code observations during the code smoothing step. Because cycle slips occur rarely, no attempt is made to connect the phase observations. For the phase observations a new ambiguity is set up at the epoch of the detected cycle slip.

## 16.1.3 Data Screening Based on Ionosphere-Free Linear Combination

When developing the program it became evident that sometimes the data was not cleaned successfully due to systematic errors in the Melbourne-Wübbena combination. These systematic errors are most likely caused by the filtering and smoothing procedures employed in the receivers. Therefore, an additional data screening step was added to the program.

In this step we build the difference between ionosphere-free linear combinations for the phase and code observations, Eqns. (9.19 and 9.20), i.e.,  $L_3 - P_3$ . As in the case of the Melbourne-Wübbena combination this linear combination should consist of noise only. The disadvantage is the amplified noise (about 3 times the noise of the  $P_1$  observations). The noise is thus about 4 times larger than the noise of the Melbourne-Wübbena combination. Nevertheless, the check is useful for removing errors caused by systematic effects. The check consists of an outlier rejection scheme, which is very similar to the one used for screening the Melbourne-Wübbena combination. The starting value for the maximum rms is larger (typically 1.6–1.8 meters), to account for the higher noise of these observations.

## 16.1.4 Code Smoothing

The final step consists of the smoothing of the code observations. The previous program steps have hopefully cleaned both, code and phase observations. This enables us now to smooth the code observations, using the carrier phase observations, for the continuous data arcs. For code smoothing it was decided to actually replace the code observations in a clean observation arc by the phase observations shifted by the mean difference code-phase in the arc. Of course we have to account for



Figure 16.2: Code residuals from point positioning. Data from a receiver installed at USNO was used for day 133 of 1999.

the opposite sign of the ionospheric effect for the code and phase observations. The smoothed code at epoch t may then be written as:

$$\tilde{P}_{1}(t) = \phi_{1}(t) + \bar{P}_{1} - \bar{\phi}_{1} + 2 \cdot \frac{f_{2}^{2}}{f_{1}^{2} - f_{2}^{2}} \cdot \left((\phi_{1}(t) - \phi_{2}(t)) - (\bar{\phi}_{1} - \bar{\phi}_{2})\right) \quad (16.1)$$

$$\tilde{P}_{2}(t) = \phi_{2}(t) + \bar{P}_{2} - \bar{\phi}_{2} + 2 \cdot \frac{f_{1}^{2}}{f_{1}^{2} - f_{2}^{2}} \cdot \left((\phi_{1}(t) - \phi_{2}(t)) - (\bar{\phi}_{1} - \bar{\phi}_{2})\right)$$

where:

$\tilde{P}_F(t)$		Smoothed code measurement at epoch $t$ and frequency $F$ .
$\phi_F(t)$		Carrier phase measurement at epoch $t$ and frequency $F$ .
$\bar{P}_F - \bar{\phi}_F$		Mean difference between all the accepted code and phase measurements in the
		current observation arc on frequency $F$ .
$\phi_1(t) - \phi_2(t)$	)	Ionospheric delay at the current epoch.
$ar{\phi}_1 - ar{\phi}_2$		Mean ionospheric delay over all the accepted phase measurements in the current
		observation arc.

Figure 16.2 shows the effect of code smoothing. Shown are the residuals of a point positioning procedure, estimating only the receiver clock offset for each observation epoch and using the CODE final orbit and satellite clock estimates. The rms error of the residuals in Figure 16.2 are 1.53 and 0.17 meters for the raw code residuals and the smoothed code residuals, respectively. The code smoothing has been quite successful. The smoothed code residuals show systematic errors of up to one meter. The size of these biases is a function of the noise of the code observations and the number of observations used in the smoothing interval. One may consider smoothed code observations as ambiguity-fixed phase observations where the ambiguities were fixed only approximately.

#### 16.1.5 Reliability and Possible Enhancements

RNXSMT has been used routinely at the AIUB since January 1997 for the estimation of receiver and satellite clock offsets using a network of 80 stations. In addition, the program was also used for all

the time transfer experiments with our two time transfer terminals. The time transfer observations are also processed routinely, with a few interrupts, since January 1997. In both applications the program proved to be very reliable and successful.

There is still room for improvement, however. One possible improvement would be to use the geometry-free combination of the phase observations for data screening and not only to correct cycle slips. Currently, data screening is based on code observations; both, the Melbourne-Wübbena and the ionosphere-free data screening, use code observations. Therefore, the data cleaning is heavily depending on the quality of the code observations. With AS turned off this is not too much of a problem, but with AS turned on the quality of the code deteriorates significantly in particular for low elevation data. It is therefore possible that a significant amount of low elevation observations is removed from the data in the outlier rejection process. The use of phase observations in the cleaning process may help to reduce the number of rejected phase observations.

Furthermore, the use of code observations also implies that small outliers in the phase cannot be detected safely. With a typical rms of 0.5 cycles for the Melbourne-Wübbena combination the  $4\sigma$  outlier detection level is about 160 cm. All (carrier phase) outliers below this level will not be detected. They will have to be detected by screening the observation residuals after a parameter estimation step.

Another improvement would consist of weighting the observations with  $\cos^2 z$ , z being the elevation angle. The clear elevation angle dependence of the data quality, in particular of code observations, could then be taken into account. Such a procedure requires additional information like, e.g., receiver and satellite positions. The current procedure requires no external input, except for the RINEX file. Another possibility to weight the observations is by using the signal to noise ratio of the observations. This information is available in the RINEX files and therefore does not require any additional external input. One big advantage of our current approach is its complete independency on the position and velocity of the receiver. This means that this data cleaning method can also be used for kinematic applications and even for very fast moving object, e.g., Low Earth Orbiting satellites (LEOs).

# 16.2 Processing Undifferenced GPS Data

The ZD processing scheme in the *Bernese GPS Software* is relatively simple. Less steps are required than for the DD processing. We start with cleaning the data on the RINEX level using the program RNXSMT (RINEX Smoothing, <u>Menu 2.7.6</u>). The cleaned RINEX data is converted to the Bernese data format (RXOBV3, <u>Menu 2.7.1</u>). After this step we are ready to perform the parameter estimation (GPSEST, <u>Menu 4.5</u>). Because the cleaning of the undifferenced data is more difficult than the cleaning of the double-differenced data we have to check and remove outliers in the residuals using the RESRMS (<u>Menu 5.3.2</u>) and SERVOBS (<u>Menu 5.1</u>) programs, see Chapter 10. Below we will describe the most important options of these five steps (RNXSMT, RXOBV3, GPSEST, RESRMS, SERVOBS). We will thereby focus on the options that are different compared to processing DD-data.

## 16.2.1 ZD Pre-Processing

The first step in the ZD processing is the cleaning of the undifferenced data. In the *Bernese GPS Software* we use the previously described program RNXSMT, Menu 2.7.6, for this purpose.

```
2.7.6
             TRANSFER: PHASE CLEANING/CODE SMOOTHING
 CAMPAIGN
                    > TIMEST42 <
                                   (blank for selection list)
Rinex Input Files:
                                   (blank for selection list)
 RINEX
                    >
                               <
Extension of Rinex Input Files (Wildcards allowed):
 EXTENSION
                      > ??0 <
Extension of Rinex Output Files (Wildcards not allowed; Overwrite Files if
 EXTENSION
                      > SMT <
                                 blank or same extension as Input Files)
```

2.7.6-1	PHASE CLEANING/CODE SMOOTHING	INPUT O	PTIONS	
Title	CLEANING UNDIFFERENCED GPS CUDE AND	PHASE D	ATA	<
_				
Detect	;ion Level:			
SAMI	PLING INTERVAL FOR RINEX DATA	> 10.0	<	seconds
MAX	IMUM GAP BEFORE STARTING A NEW ARC	> 180.0	<	seconds
MAX	IMUM GAP FOR CYCLE SLIP CORRECTION	> 180.0	<	seconds
RMS	OF MELBOURNE WUBBENA	> 0.60	<	L5 cycle
RMS	OF L4 FOR FIT AND CYCLE SLIP CORR.	> 0.01	<	meters
RMS	OF L3-P3	> 2.0	<	meters
STZE	E OF DETECTABLE CYCLE SLIPS	> 1.0	<	L5 cvcles
SIZE	E OF DETECTABLE OUTLIERS	> 5.0	<	L5 cvcles
				<i>,</i>
Cvclv	Slip Options:			
MINI	IMUM NUMBER OF OBSERVATIONS IN AN ARC	> 10	<	
NUME	REP. OF 14 ORSERVATIONS FOR FIT	> 10	Ż	
ETVO	CYCLE CITES IN CODE ODGEDUATIONS	VEC	2	(VEC NO)
	DICLE SLIPS IN CODE UDSERVAIIUNS	> 1E5		(115, 10)
FLAC	AD PHASE UBSERVAIIUNS	> ies	<	(YES, NU)
PRI	NT OPTION	> SUM	<	(SUM, ALL)

There are only two input panels for RNXSMT. In the first panel the input RINEX files are selected and the extension of the output RINEX files. It is possible to overwrite the original RINEX files but this is not recommended! In the second panel (Panel 2.7.6–1) the cleaning options are specified. Because the program was described in detail in the previous section we refer to this section and to the help panel for a detailed description of the input options.

In the second ZD processing step we have to convert the clean RINEX files to the Bernese observation file format using RXOBV3 which was described in Chapter 10. We now have to use the special RXOBV3 options in the menu system. For bad observations RNXSMT has set the signal to noise ratio of the observations to 1 and it has set the cycle slip flag at those points where it has detected a cycle slip. Normally, we do not use the flags in the RINEX files when converting RINEX data to the Bernese format. Now, after having cleaned and flagged the RINEX data, we do want to use this information. For this purpose we have to set the options in Panel 2.7.1–1 to accept the information from the RINEX files. We therefore have to set the "MINIMUM SIGNAL STRENGTH" to "2", "ACCEPT SIGNAL STRENGTH = 0" to "NO", and "ACCEPT CYCLE SLIP FLAGS FROM RINEX" to "YES" as shown below.

2.7.1-1	RINEX OBS.: INPUT						
Title L: TITLE	ine: > RXOBV3 AFTER RNXSMT				<		
Signal S MINIMU ACCEPT ACCEPT	Strength Requirements: JM SIGNAL STRENGTH F SIGNAL STRENGTH = 0 F CYCLE SLIP FLAGS FROM RINEX	> > >	2 NO YES	< < <	(0-9) (YES or NO) (YES or NO)		
Sampling SAMPL SAMPL LIMIT	g: ING INTERVAL ING OFFSET TO FULL MINUTE DATA TO SESSION DEFINITION	> > >	0 YES	< < <	(sec; blank: take all obs) (sec) (YES or NO)		
Session LENGTH	Numbering: 4 OF SESSION NUMBERS	:	> 4 ·	<	(3 or 4 characters)		

After converting the data into the Bernese observation file format we are ready to start processing the ZD data. Of course, just like in the DD processing, we do need some other a priori information like station coordinates, satellite orbits, etc. Helpful, but not mandatory, a priori information in the case of ZD processing is a priori satellite clock information.

# 16.2.2 ZD Processing

If we have cleaned the zero-difference observations and converted them to the Bernese observation file format and if we have prepared all other required a priori information, like orbits and coordinates, we can use GPSEST to process the data. Most of the options are identical with the normal DD processing as discussed in the previous chapters. We will therefore only discuss those options which are special for the ZD estimation. First of all we have to select "Z.DIFF" (ZD) instead of "S.DIFF" (SD) observation files in Panel 4.5. In the ZD processing we will use both code and phase observations simultaneously, contrary to DD processing where we use only phase observation. In the DD processing the code data is only used for the clock alignment in the CODSPP step. In principle, if you are not interested in the clock estimates, you can also choose to process only the phase observations. This may be useful if you have really bad code observations but this is not commonly the case, so we advise to use the code and phase data simultaneously. Besides the selection of ZD files we can also specify the a priori satellite clock information here. Since this is usually available, either from broadcast or from the IGS precise orbits, we advise to specify it here although it is certainly not mandatory.

4.5	PROCESS	SING:	: PARAMETER ESTIMATION
CAMPAIGN	> TIMEST42	2 <	(blank for selection list)
Job Identifica JOB CHARACT	ation: ER > <		(blank, or AZ, O9)
Input Files: PHASE Z.DIFI CODE Z.DIFI PHASE S.DIFI CODE S.DIFI COORDINATES STANDARD OR: RAD.PRESS.CI IONOSP. MODI TROPO. ESTII METEO DATA ECCENTRICIT: OCEAN LOADII SATELL. CLO CODE BIASES ANT. ORIENT.	F. > %%%%%%%%% F. > %%%%%%%%% F. > NO F. > NO BIT > CL_96168 DE. > NO ELS > NO MATES > NO IES > NO IES > NO NG > ITRFCODE CKS > CL_96168 > NO ATION > NO		<pre>(NO, if not used; blank for sel.list) (NO, if not used; blank for sel.list) (NO, if not used; blank for sel.list) (NO, if not used; blank for sel.list) (blank for selection list) (blank for selection list) (NO, if not used; blank for sel.list) (NO, if not used; blank for sel.list)</pre>

In the output panel (Panel 4.5–0) there are two clock output files, one satellite clock output file and a clock-RINEX output file. The satellite clock output file will be generated in the "ORB" subdirectory of your campaign whereas the clock-RINEX output file will be generated in the "OUT" subdirectory of your campaign. The extension of both file types is "CLK" but because they are generated in different directories you can give them the same name. The clock-RINEX file will contain both the station and satellite clock information. So the information in the satellite clock file is in principle redundant. However, the clock-RINEX can not yet be used in the Bernese software which is why we still support the satellite clock information file. If you want to obtain the clock estimates (and residuals) you have to specify a residual output file in Panel 4.5–0.

If you have an early distribution of the *Bernese GPS Software*, Version 4.2 it is possible that you miss the output for the clock RINEX file. You may add this option when you update your software. The files are necessary for this step may be found at our ftp server:

ftp :	ftp ubecx.unibe.ch						
login:	anonymous						
password:	your full e-mail address						
location:	cd aiub						
	cd BERN42						
	cd SOURCE						
	cd DOS or cd UNIX						
	cd CLKRNX						

You will find a file "README.CLKRINEX" in this directory. It describes the steps for the update on your platform in detail.

4.5-0	]	PAF	R. ESTIMAT	CION:	OUTPO	UT I	FILES	3				
0	- <b>P</b> : 1											
outpu	IT FILES:		02 06165	/	( NO	: e	<b>n</b> +	+ -	ha			
	JEDINALES	(	NO NO	2	(NO,	11 : 4	not	10	be	saved)		
	DITAL ELEMENTS	(	NU C2 06165	2	(NO,	11 : 4	not	10	be	saved)		
	DOCDUEDE CRADI	(	NO NO	2	(NO,	11 : 4	not	10	be	saved)		
	DOCDUEDE CINEX	~	NO	~	(NU,	11	not	το	be	saved)		
	JPUSPHERE SINEA	~	NO	~	(NU,	11	not	το	be	saved)		
TOL	NUSPHERE MUDELS	~	NU		(NU,	11	not	το	be	saved)		
101	IUSPHERE MAPS	~	NU GO OGICE	<	(NU,	11	not	to	be	saved)		
RES	ADIANS	~	C3_96165	<	(NU,	11	not	to	be	saved)		
CUN	ARIANCES (CUURD)	>	NU	<	(NU,	11	not	to	be	saved)		
CUN	ARIANCES (ALL)	>	NU	<	(NU,	11	not	to	be	saved)		
NOF	AMAL EQUATIONS	>	C3_96165	<	(NO,	1Ť	not	to	be	saved)		
EAF	TH ROTATION PARA.	>	NO	<	(NO,	if	not	to	be	saved)		
POI	LE IN IERS FORMAT	>	NO	<	(NO,	if	not	to	be	saved)		
SAT	TELLITE CLOCK FILE	>	C3 <u>9</u> 6165	<	(NO,	if	not	to	be	saved)		
CLC	CK RINEX FILE	>	C3_96165	<	(NO,	if	not	to	be	saved)		
COI	DE BIASES	>	NO	<	(NO,	if	not	to	be	saved)		
ANT	TENNA PCV (GRID)	>	NO	<	(NO,	if	not	to	be	saved)		
ANT	TENNA PCV (HARM)	>	NO	<	(NO,	if	not	to	be	saved)		
GEN	IERAL OUTPUT	>	C3 <u>9</u> 6165	<	(NO,	if	star	ndaı	rd i	name to	be	used)

When processing ZD data, we can not resolve ambiguities. So the options of introduction and saving of ambiguities (Panel 4.5–1) are obsolete. If you want to obtain clock estimates and/or residuals the ambiguities are not to be pre-eliminated. In the ZD case normalized residuals may be computed incorrectly. The residuals are slightly pessimistic especially if the epoch solution is nearly singular. We therefore advise not to compute normalized residuals (Panel 4.5–0.1).

The ZD observations must be processed simultaneously. Therefore the "CORRELATIONS" option in Panel 4.5–2 must be set to "CORRECT". Of course, this has a completely different meaning than in the case of the DD processing. Furthermore, in this panel we may specify that we want to perform ZD processing. Although GPSEST will automatically do ZD processing in case ZD files are introduced, this option gives access to some additional input option panels.

4.5-2		PARAMETER ESTIMATION: INPUT 2										
Atmosp METE	ohere Models: 20 DATA	> E	EXTRAPOLA	TED	<	(EXTRAPOLATED, OBSERVED						
TROF	POSPH. MODEL	> 1	10		<	or ESTIMATED) (SAASTAMOINEN,HOPFIELD, ESSEN-FROOME,MARINI-MUR, DRY SAAST,DRY HOPFIELD, or NO)						
Statis CORF CORF A PF ELEV	stics: RELATIONS REL. INTERVAL RIORI SIGMA 7DEP. WEIGHTING	> (	CORRECT > 1 > 0.001 > NO	< < <	<	(CORRECT, FREQUENCY, or BASELINE) sec m (NO, COSZ, or model number)						
Furthe PRIN HELN ORBJ SPEC ZERC	er Options: MING MERT IT ADJUSTMENT DIAL REQUESTS D DIFFERENCE EST.		> NO > NO > NO > YES > YES	< < < < <		(YES NO or ASIS) (YES NO or ASIS) (YES NO or ASIS) (YES or NO) (YES or NO)						

In Panel 4.5–2.4, we have to set the "PARAMETER PRE-ELIMINATION" option to "YES". All other options have their normal meaning. In Panel 4.5–2.4.8, we then have to specify the epoch wise ("EP") pre-elimination of the satellite and station clocks. Of course the necessity of the pre-elimination of these parameters depends on how large your parameter estimation matrix dimension

in GPSEST is. But the number of clock parameters tends to be rather large! For each satellite and station one clock parameter is estimated for each epoch. With 8 stations observing on average 12 satellites this gives 20 parameters per epoch. With a 5 minute sampling rate, 288 epochs, this yields 5760 parameters to be estimated for one day. The number of parameters we currently use in our IGS processing is only 2000. Therefore, under normal conditions, the clock parameters will be pre-eliminated each epoch. This also speeds up the processing enormously because the matrices remain relatively small. Contrary to all other pre-eliminated parameters it is possible to obtain clock estimates after they have been pre-eliminated. To obtain clock estimates and observation residuals after the clock parameters have been pre-eliminated, make sure that you have specified the output of a residual file in Panel 4.5–0. The clock estimates and the observation residuals will then be computed using a back substitution of the other solved-for parameters. Note that in this case the phase ambiguities may not be pre-eliminated.

4.5-2.4.8				PAR	AMETER	PRE –	ELI	MINAT	C I C	DN					
Paramet	ers to be Pre	ə-H	Elin	linated	1: NO= BI= AI= EP=	No P Befo Afte Afte	re- re r r E	Elimi Invei Invei ach H	ina rsi Epo	atic ion ion och	on of of	Norma: Norma:	L Eq L Eq	Sy: Sy:	stem stem
STATI RECEI ORBIT AMBIG REC.H SITE LOCAL DIFF. LOCAL EARTH STOCH SAT.	ON COORD. VER CLOCKS ELEMENTS UITIES EIGHT.CALIB. TROPOSPHERE CODE BIASES TROPOSPHERE ROTATION CORBIT ANT.OFF	> > > > > > > > > > > > > > > > > > > >	NO NO NO NO NO NO NO NO	< < < < < < < < < < < < < < < < < < < <	EARTH HILL F EARTH CENTEF DIFF. PHASE GLOBAI KIN. ( STATIC SATELI	POTE RESON ALBE OF IONO CENT ION COORD DN CL LITE	NTI DO MAS SPH ER OSP INA OCK CLO	AL ES ERE VAR. HERE TES S CKS	>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>	NO NO NO NO NO NO NO EP EP	~~~~~~~~	(NO, (NO, (NO, (NO, (NO, (NO, (NO, (NO,	BI, BI, BI, BI, BI, BI, BI, BI, BI, BI,	AI) AI) AI) AI, AI, AI) AI) AI, AI, AI,	EP) EP) EP) EP)

In Panel 4.5–3, the clock estimation is defined. When processing ZD GPS observations it is necessary to estimate the satellite and receiver clock offsets. Remember that in the DD case one has cancelled the effect of the clocks by double-differencing the observations. Herewith one implicitly estimates the satellite and receiver clocks as white noise parameters. In the ZD case we have to explicitly estimate the clocks. However, it is only possible to solve the clocks in a differential manner. All but one clock may be estimated, i.e, either a receiver or a satellite clock has to be kept fixed. In the menu system, we mainly support the selection of a reference receiver clock, but it is also possible to use one of the satellites as reference, but this can only be done by manually editing the input option file (I-file). In theory, it is also possible to fix a kind of a "mean clock", e.g., specifying that the sum of the clock offsets of a number of receivers should be zero. This, however, has not yet been implemented in our software. At the moment it is requested that observations for the selected reference clock are available for each epoch; otherwise no clock estimates are available for those epochs where the data of the reference clock are missing. For the "normal" parameters, the quality of the reference clock is not important; the only prerequisite is that it should be aligned to GPS time at the millisecond level, for the same reason as in the DD case. If, however, you are interested in the clock estimates, it is wise to select a receiver which is connected to a stable external oscillator as reference clock. The selection of a reference station can be done using a "SPECIAL\_FILE" just like for the coordinates. However, you should make sure that only one station is specified in such a "FIX" file, and that data of this station is actually available for the day you are processing. The selection of the name of the special file is done in Panel 4.5–3.1. The "O-C EDIT LEVEL" is of no importance, please leave it to "0.0". This option is mainly available for some special test cases.

```
      4.5-3
      PARAMETER ESTIMATION: INPUT CLOCK ESTIMATION

      CLOCK ESTIMATION:
      REF.STATIONS > SPECIAL_FILE

      REF.STATIONS > SPECIAL_FILE
      < (blank for sel.list, ALL, NONE
SPECIAL_FILE, $FIRST, $LAST)

      SATELLITES > ALL
      < (ALL or NONE)</td>

      ESTIMATION OPTIONS:
      O-C EDIT LEVEL
      > 0.0

      O-C EDIT LEVEL
      > 0.0
      < m (0.0 for no editting)</td>
```

With the options specified in this way, you are all set to process ZD data with GPSEST. In the general output file, you should then find an output block in the test output area looking similar to what is shown in Figure 16.2.2. Notice that releases of *Bernese GPS Software* Version 4.2 dated before December 2000 will show a somewhat different output. This output gives the clock estimates for each epoch. The station clock estimate lines start with the number 23 which is the internal parameter type number in our software. The satellite clock estimate lines start with 24. The second number on the line for the stations is the internal station number and is not important. For the satellites, this number reflects the PRN code. The next field gives the time in MJD, followed by the clock estimate. For the stations, this field reflects only the estimate. For the station is not available when solving for the estimates. Therefore, we advise you to work without a priori clock information for the stations. This means that you should **not** run CODSPP prior to running GPSEST in the case of ZD processing. In the next release, the handling of a priori clock information will be solved. The

23	5	50247.000000	0.00000	0.00000	#	WTZR	14201M010	
23	1	50247.000000	5.311692	7.024650	#	BRUS	13101M004	
23	3	50247.000000	4.537303	7.024650	#	KOSG	13504M003	
23	4	50247.000000	-0.616983	7.024671	#	ONSA	10402M004	
24	2	50247.000000	-264.160688	6.570986				
24	16	50247.000000	14.529693	6.570981				
24	18	50247.000000	4.000320	12.167082				
24	19	50247.000000	473.206547	6.570977				
24	26	50247.000000	-210.969011	702.493778				
24	27	50247.000000	31.555178	6.570981				
23	5	50247.002083	0.00000	0.00000	#	WTZR	14201M010	
23	1	50247.002083	5.311675	7.024651	#	BRUS	13101M004	
23	2	50247.002083	108.672666	6.846803	#	JOZE	12204M001	
23	3	50247.002083	4.537493	7.024650	#	KOSG	13504M003	
23	4	50247.002083	-0.617150	6.846804	#	ONSA	10402M004	
23	6	50247.002083	211.266854	7.024672	#	ZIMM	14001M004	
24	2	50247.002083	-264.179545	6.083552				
24	16	50247.002083	14.687344	6.083547				
24	18	50247.002083	4.057994	9.292760				
24	19	50247.002083	473.235579	6.083541				
24	26	50247.002083	-210.931732	702.493778				
24	27	50247.002083	31.600691	6.083546				

# Figure 16.3: Clock output block in GPSEST general output file, clock estimates in microseconds, formal errors in nanoseconds.

clock estimates are given in microseconds. Following the clock estimates field is the formal error of the estimates which is given in nanoseconds. The "0.000000" for the station "WTZR" indicates that this was the station which was selected as reference clock. Note, that these formal errors are not scaled with the rms of the solution! Furthermore, the formal errors are not computed correctly. In the "back-substitution" process, the solved for parameters (e.g., ambiguities, troposphere, station coordinates) are introduced as known values. Therefore, the covariance matrix is incorrectly yielding (much) too optimistic formal errors. This is also the reason why the normalized residuals are not computed correctly. For the computation of the normalized residuals, we need the covariance matrix. Since this is incorrect, the normalized residuals are not correctly computed either. However, these normalized residuals will be too large, so you are always on the "safe side". For the station clock estimates the last column lists the name of the station as found in the coordinate file.

After running GPSEST in this way and obtaining clock estimates and residuals, RESRMS may be used to screen the residuals for outliers. Here one must keep in mind that RNXSMT is only capable of screening the data at the meter level. This means that small cycle slips and small outliers in the phase observations may have gone by unnoticed. Therefore in the ZD processing case we perform at least two iterations to clean the data with two different residual levels. In the DD case we also use multiple iterations but all with the same screening window. For the ZD case, using real residuals, the residual level (Panel 5.3.2–1) is set to 0.100 and 0.020 m for the first and second iteration respectively. When using normalized residuals and elevation dependent weighting, we use 0.025 and 0.006 m.

After RESRMS, we can use SERVOBS (<u>Menu 5.1</u>) or SATMRK to remove the outlier residuals from the actual observation files. If one is interested in clock estimates, it is wise to not only remove the phase observations but also the corresponding code observations. This to avoid that clock estimates for certain epochs are based on code observations, only. Currently, this is not handled automatically by the software. Some manual intervention is needed in the "EDT" file. After removing the outlier observations, the next GPSEST iteration can be run. After the second outlier removal, the third GPSEST run can be performed.

# 16.2.3 Precise Point Positioning

A special application of the ZD processing approach is the so-called precise point positioning (PPP). The main difference here is that instead of estimating the satellite clocks, the satellite clock information is taken from an external source like, e.g., the IGS products. This allows to process data of one single station using IGS orbits and clocks. Of course one is not limited to processing one single station, several stations may be processed simultaneously but they will have no correlation with each other unless some common parameters are estimated.

For PPP, it is very important that the orbit and clock information are and remain consistent. This means that the orbit and clock information should come from the same source and that the orbit is represented at the mm level. As we have seen in the example (see Chapter 4), ORBGEN can fit the IGS orbits at the few cm level. This is not good enough for PPP! For the PPP application it is necessary to estimate multiple arcs for every 24 hour interval to ensure that the orbit is represented at the mm level. We usually get good results using 4 arcs for a 24 hour interval, each arc being 6 hours long. This estimation of 4 arcs can be achieved by setting the "# OF ARCS" to "4" in Panel 3.3–1. In each arc, the satellite motion should be modeled using the full set of available orbit and radiation pressure parameters (15 parameters in total). Furthermore, the clocks should be used as provided,

i.e., without interpolation. Note, that this implies that one should process the data with a sampling rate compatible to the sampling rate of the satellite clock information. When using precise orbit files as input, this means that you can use a sampling rate of 15 min, only. This is a significant limitation of the PPP approach. Furthermore, the models used when generating the orbits and the clocks should be compatible to the models used in the software performing the PPP. Amongst the GPS software packages used within the IGS, this should pose no problems. Results from other GPS software packages may not be compatible though.

Apart from the small change in generating the standard orbit the rest of the processing options remains the same as above, except for the selection of the clock estimation in Panel 4.5–3. Here we now have to specify "NONE" for both the "REF.STATIONS" and the "SATELLITES". This means that all station clocks and no satellite clocks will be estimated. The resulting coordinate estimates should be as good as an ambiguity free network solution. This quality should be good enough for most GPS applications. Note that because PPP is similar to processing the station in question in a global network, it is always necessary to estimate tropospheric delay parameters. This is also the case when performing PPP on two very nearby stations, for both stations tropospheric zenith delay parameters have to be estimated.

One of the main disadvantages of the PPP is that one can not do ambiguity resolution. Furthermore, the relative information between the stations is lost because the correlations between the stations are unavailable. On short baselines, this gives a significant degradation of the baseline estimates. For regional solutions the loss of the covariance information makes the "stacking" of solutions using normal equations impossible. Nevertheless, PPP is a very efficient method to get high accuracy results.

Point positioning may also be done using only code observations. In this case one can choose between CODSPP and GPSEST. Both programs should give more or less the same quality of results  $(\pm 50 \text{ cm})$  when using code observations, only. However, we discourage the use of CODSPP for this purpose because this program will save the receiver clock estimates in the observation files. Because of the 15 min. sampling this renders the files unusable for further processing using the DD approach. Therefore, we strongly advise you to use only GPSEST for precise point positioning. CODSPP should be used for the clock alignment only, without using any sampling, i.e., estimating a clock offset for each epoch!

# 16.2.4 Processing SLR Data

Another special application of the ZD processing approach is the processing of Satellite Laser Ranging (SLR) data. The enhancement of GPSEST to handle undifferenced GPS data made it very simple to also allow the processing of SLR data. For this purpose we have a special program which converts the SLR data from the so-called "quick-look" (QL) format to the RINEX format. This program (QLRINEXO) is, however, not provided with the software but is available on request. Here we will give only a very brief overview of how to process SLR data in the Bernese software because this is something which is mainly interesting for scientific purposes and will not be too widely used.

In converting the SLR data to the RINEX format the SLR ranges are divided by two to make them "one-way" ranges, which is better compatible with the GPS range observations. In the same step also the meteorological observations, as available in the QL files, are converted to RINEX meteo files. The SLR RINEX observation and meteo files are converted to their Bernese formats using RXOBV3 and RXMBV3. In the RXOBV3 step one must set the "RANGES" option in Panel 2.7.1

to "YES". In this way RXOBV3 will generate only code zero-difference files. In GPSEST one should select these code zero-difference files and the Bernese meteo files. Since in the case of SLR clock estimation is not necessary we set (leave) the "ZERO DIFFERENCE EST." to "NO" in Panel 4.5–2. Also, when processing SLR data one should use the "MARINI-MUR" option for the "TROPOSPH. MODEL". Contrary to the GPS (microwave) signals, the SLR (optical) signals are only influenced by the hydrostatic part of the atmosphere. If local meteorological measurements are available, the tropospheric effect on the SLR measurements can accurately be modeled using the Marini-Murray model. This is the reason why the meteo files are needed when processing SLR data in GPSEST. In this way, there is no further need to estimate tropospheric zenith delay parameters.

Currently, the primary usage of SLR data is to verify the quality of our GPS and GLONASS orbits. For this case no parameters are actually estimated. Our CODE orbits are introduced as "truth", as are the SLR measurements. The residuals of the SLR measurements after modeling them in GPSEST gives us a good idea of the absolute quality of our GPS and GLONASS orbits. For GPS, the number of SLR observations is too small to do anything else with them. For GLONASS, there is enough data to do actual orbit determination using only SLR data, or combined SLR/microwave processing. Some examples of the SLR work done with the *Bernese GPS Software* may be found in [*Springer*, 1999].

In our opinion the combined processing of the IGS data with the SLR data from the two Lageos satellites (altitude of 6000 km) will be a very interesting research topic for the next years. The importance of the SLR processing capability will be further enhanced by the upcoming Low Earth Orbiter (LEO) missions which will carry both GPS receivers and SLR retro-reflectors.

# 16.3 Example of Processing Undifferenced Data

To give you some help in understanding how the ZD processing works, we have generated a BPE example which does a simple ZD processing for you. This example uses the same data as the regional processing example discussed in Chapter 4. The actual processing is very similar, the basic differences is that we now use undifferenced data. Besides the description in the manual the example from Chapter 4 also comes with a "normal" (double-difference) BPE example, see Chapter 22. Both examples should be in your user BPE environment (U:/PCF) after successfully installing the software. The DD example is called DOCU42\_1.PCF whereas the ZD example is called TIMEST42.PCF. The accompanying script and option directories should also have been installed automatically when setting up your user environment (U:/SCRIPT, U:/OPT/...). Note, however, that this Chapter and also the BPE example were written after releasing the *Bernese GPS Software* Version 4.2. So the ZD BPE example is only available since December 2000. If you got Version 4.2 earlier than this date you will not find the example. If you are interested in this BPE example you may download it from our ftp server:

ftp :	ftp ubecx.unibe.ch
login:	anonymous
password:	your full e-mail address
location:	cd aiub
	cd BSWUSER
	cd EXAMPLES
	cd UNDIFF42

You will find a file named README.TXT with further instructions.

The ZD BPE example basically performs the processing steps described in the previous sections. Figure 16.4 shows the processing steps made in the BPE example.

PID	SCRIPT	OPT_DIR	CAMPAIGN	CPU	Ρ	WAIT	r FOF	R							
3**	8******	8******	8******	8******	1	3**	3**	3**	3**	3**	3**	3**	3**	3**	
001	DOCU_COP	TIMEST42		any	1										
002	PRETAB	TIMEST42		any	1	001									
003	ORBGEN	TIMEST42		any	1	002									
004	RNXSMT	TIMEST42		any	1	001									
005	RXOBV3	TIMEST42		any	1	004									
006	TIMEST	TIMEST42		any	1	003	005								
007	TIMEST	TIMES242		any	1	006									
008	TIMEST	TIMES342		any	1	007									
009	DOCU_DEL	TIMEST42		any	1	008									

Figure 16.4: BPE processing steps for undifferenced processing example.

The first script "DOCU\_COP" copies some data files to their appropriate directories and names. The next two steps (PRETAB and ORBGEN) generate the a priori standard orbit based on the input precise orbit file. Step 004 (RNXSMT) performs the zero-difference code smoothing and data cleaning, and step 005 converts the clean RINEX data into the Bernese format (RXOBV3).

Steps 006, 007, and 008 are the two iterations and the final estimation of the undifferenced solution. Each "TIMEST" script executes the program chain GPSEST, RESRMS, and SERVOBS. The RESRMS residual level differs in step 006 and 007, but the options for the parameter estimation remain the same for all three executions of GPSEST. Only in the final execution (step 008) more output files are saved. Even in the third step, the residuals are screened and outliers are removed. Of course this has no effect on the solution, but is merely done for ease of implementation, i.e., using an already existing script. However, it allows to verify if the data is really clean, i.e., if there are still outliers after the last (final) run. The results of the processing are summarized in the "OUT" subdirectory of your campaign and are called "EXAMPLE.96\_165" and "EXAMPLE.96\_166" for the two days of the example. They basically contain the summary of the ORBGEN fit, the residual statistics of the three RESRMS runs, and a (hopefully small) list of remaining outliers from the third residual screening. The last entries are the summaries of the three GPSEST runs. Here it is especially interesting to check how large the number of observations differs between the first and the last solution. The rms of the GPSEST is not very reliable. This is caused by numerical problems in the computation of the rms, due to the bad initialization of the phase ambiguities in the ZD case. We are working on resolving these problems in our current version of the software. The real rms may be found in the RESRMS residual statistic. Problematic is, however, the fact that the rms is based on the code and phase observations where the code observations have been scaled by a factor of 1000.

If you are planning to use the ZD capabilities of the *Bernese GPS Software*, please take the time to run and understand the steps taken in this example BPE. Many things may be unclear from just reading this documentation. The example should help you to understand all the information given above.

# 16.4 Clock Files in the *Bernese GPS Software*

# 16.4.1 Satellite Clocks

Satellite clock information is available either in the RINEX navigation data, in the precise orbit (SP3) file format, or in the clock-RINEX format. These formats have to be converted to a format suitable for the *Bernese GPS Software*. The RINEX navigation file has to be converted to the Bernese broadcast file format by the program RXNBV3, see Chapter 7 and Panel 2.7.2. This broadcast format contains both the satellite orbit and clock information. A separate clock file can be generated from the Bernese broadcast file using the SATCLK program, Panel 3.8. The SP3 file format can be converted with the program PRETAB, see Chapter 8. This program generates two separate files, one for the satellite orbit and one for the clock information. The orbits will be in the Bernese tabular format and the clocks in the Bernese satellite clock format. For the clock-RINEX format we have currently no converter available nor can the clock values be read by any of the current *Bernese GPS Software* programs. For a description of the different file formats we refer to Chapter 24.

Satellite clocks are usually modeled by a low degree (1 or 2) polynomial. Therefore, when generating a clock file with program PRETAB, the user is requested to enter a title for the clock file, an interval length, and a polynomial degree. We recommend to use a polynomial degree of 2 for each 12-hour interval.

To make full use of the precise clock information in the precise orbit format (SP3), i.e., for PPP, we will have to run PRETAB with special options, to generate a satellite clock file containing the actual values from the precise orbit file and not a polynomial fit. For this purpose the polynomial degree and interval should both be set to zero ("0") Panel 3.2–1. The resulting satellite clock file will contain only satellite clock information every 15 minutes, because this is the standard IGS sampling rate in the SP3 files. This clock file should only be used for precise point positioning!

# 16.4.2 Receiver Clocks

Receiver clock information is available in the clock-RINEX files. However, the *Bernese GPS Software* currently has no converter available nor can the clock values be read by any of the available programs. Receiver clock information is thus not readily available and consequently has to be estimated prior to forming single or double differences. For the ZD approach a priori clock information is not necessary. In the *Bernese GPS Software*, this is done by the program CODSPP, see Chapter 10. This program performs a single point positioning using only code observations. For this step satellite clocks are required! Because satellite clocks are mandatory in CODSPP, the program can read both file formats containing satellite clock information, the broadcast as well as the satellite clock file formats.

The single point positioning step allows the microsecond level synchronization of the receivers with respect to GPS time and with respect to each other. This alignment to GPS time is achieved by using the satellite clock information. The estimated receiver clock corrections should be saved in both the code and phase zero-difference observation files.

Under SA (Selective Availability), mainly due to the clock dithering, the rms of the single point positioning will be of the order of 25 meters only. Without SA the rms will be around 5 meters. Even large rms values of up to 300 meters should cause no real problems later on in the processing,

because 300 meters correspond to 1 microsecond. Nevertheless, one should have a careful look at the results.

Within the IGS, there are several analysis centers providing satellite clock estimates. These estimates are, however, in most cases not aligned to GPS time, but rather to one particular reference clock. Only the clock estimates of CODE and the combined IGS clocks are aligned to GPS time. The clock offset differences between GPS time and a particular reference clock may be as large as 1 microsecond. This is something one should be aware of when using satellite clock information from different sources!

## 16.4.3 Clock Input in Other Programs

In addition to CODSPP, it is also possible to use the satellite clocks in the programs MAUPRP and GPSEST, but only in the satellite clock format. This possibility was created because in the early days of GPS not all receivers did measure near the full second. Therefore, observation times could differ by hundreds of milliseconds. To be able to process double-differences corresponding to different epochs, one has to account for the satellite clock drift between "simultaneous" epochs. Of course, this only works if SA is not turned on. Nowadays, SA is permanently turned off and fortunately the receivers are all measuring within one millisecond of the full GPS second. So, although the possibility exists to use satellite clock information during MAUPRP and GPSEST processing, this is not really necessary. Of course for the ZD approach the availability of a priori clock information is useful, but not mandatory, and we advise using the available a priori satellite clock information.

Because the receiver clocks are written into the zero-difference observation files by the program CODSPP, this program has to be run for the DD processing approach. The receiver clock corrections are passed from the zero-difference files to the single-difference files when forming baselines with the program SNGDIF, <u>Menu 3.4</u> and are then automatically used in all programs processing the SD observations. For a description of the observation files and the clock corrections please refer to Chapter 24. In the ZD approach, one may run CODSPP prior to running GPSEST. However, GPSEST only outputs the receiver clock changes and can not output the "total" clock offset. If you are interested in the receiver clocks, it is best **not** to run CODSPP prior to the clock estimation step. In the next release of the software, this problem will be resolved.

# 17. Antenna Phase Center Offsets and Variations

# 17.1 Motivation

In Chapter 9, we stated that GPS observations may be modeled by Eqns. (9.14). Neglecting the clock corrections and atmospheric biases (which are not relevant for the topic of this chapter), we have an observation equation which (in the case of phase measurements) looks like

$$L^i_{Fk} = \varrho^i_k + \lambda_F \ n^i_{Fk} \tag{17.1}$$

substituting F for the frequency index. The term  $\varrho_k^i$  was declared to be "the geometrical distance between satellite *i* (at signal emission time) and receiver k (at signal reception time t)". However, if highest accuracy is required, this definition is not sufficient. We have to specify the exact points with respect to which the geometrical distance  $\varrho_k^i$  should be measured. These points are called "antenna phase centers".

# 17.2 Satellite Antenna Phase Centers

The precise position of the phase center of the satellite transmitting antenna with respect to the center of mass of the satellite is given in file X:/GEN/SATELLIT.TTT, which has to be specified in Panel 0.3.1 (see also Chapter 24):

```
      SATELLITE-SPECIFIC DATA FOR GPS AND GLONASS - VERSION 4.2
      29-JAN-01

      RADIATION PRESSURE MODEL : T980301
      (ROCK MODEL T, FLIEGEL ET AL, 1992)

      BLOCK ANTENNA OFFSETS (M) NASS DPO P2 ROCK MODEL
      PRN NO. DX DY DZ (KG) (1.E-8) (1.E-9) (T=1,S=2)

      1
      3
      0.2794
      0.0000
      1.0230
      975.0
      -0.1088
      0.7458
      1
      0.0000

      2
      2
      0.2794
      0.0000
      1.0230
      880.0
      -0.0373
      0.6362
      1
      0.0000

      3
      3
      0.2794
      0.0000
      1.0230
      975.0
      -0.0395
      0.5637
      1
      0.0000
```

It is assumed that the phase center is the same for the  $L_1$  and  $L_2$  carriers (and therefore also for all linear combinations of  $L_1$  and  $L_2$ ) and that the phase center location remains constant. We will see that the location of the *receiver antenna phase center* depends on the *direction* the signal is coming

from. For the satellite antenna, we so far do not need to consider any possible directional dependence since there is no information available and the angle of the cone of rays is relatively acute. The angle between rays and the satellite's nadir may reach up to 14 degrees (at low elevations). There is, however, some evidence, that directional effects do exist, and they will eventually have to be corrected.

The consideration of satellite antenna offsets from the file X:/GEN/SATELLIT.TTT is of importance in the program GPSEST for reducing the range measurements with respect to the standard orbit information, which is always referred to the satellites' center of mass. In the program ORBGEN, the use of satellite antenna offsets only makes sense if the satellite positions (given in precise orbit file \*.PRE) are referred to the particular satellite antennas. The user has to specify this in <u>Panel 3.3–1</u>. The Bernese, as well as the IGS orbital information, are always referred to the center of mass of the satellites.

If the satellite passes through the Earth's shadow (for up to 55 minutes), it cannot orient itself correctly with respect to the Sun and the resulting mis-orientation of the satellite leads to biases in the geometrical distance (up to about 10 cm for very long baselines). There is also a dynamical effect caused by this mis-orientation: the solar-panels-axis is no longer perpendicular to the direction Sun-satellite and the solar radiation pressure force becomes very difficult to model. Both effects, the geometrical and the dynamical, are *not* taken into account by the software. The estimation of satellite antenna offsets (Panel 4.5-2.4.9) may be used to study the satellite antenna orientation in more detail.

To apply the antenna phase center positions from the file X:/GEN/SATELLIT.TTT correctly, it is necessary to know if the satellite positions (given in precise orbit files \* . PRE) represent the positions of the center of mass (which is the normal case) or directly the positions of the antenna phase center. The user has to specify this in Panel 3.3–1 when running program ORBGEN. The Bernese and the IGS orbits always refer to the center of mass of the satellite.

# 17.3 Receiver Antenna Phase Centers and Their Variations

In the case of receiver antennas, the situation is more complicated because the GPS signals come from different directions (from different satellites) and the position of the antenna phase center depends on this direction. This directional dependence is what we call *antenna phase center variations*. We also have to take into account that the antenna phase center positions and the antenna phase center variations are not identical for  $L_1$  and  $L_2$  carriers. Because the antenna phase center variations are antenna-type-dependent, this effect has to be carefully modeled if different antenna types are used simultaneously. Mainly the relative station height is affected. The bias may reach values up to 10 cm, *independent* of the baseline length. If only antennas of the same type are used, the main effect is a scale factor in the network of up to about 0.015 ppm due to the fact that for long distances the antennas "see" the same satellite under different elevation angles.

In the *Bernese GPS Software*, we use the following antenna phase center variation correction scheme:

$$\Delta\phi(\alpha, z) = \Delta\phi'(\alpha, z) + \Delta \mathbf{r} \cdot \mathbf{e} , \qquad (17.2)$$

where

 $\Delta \phi(\alpha,z)$  ... is the total phase center correction in the direction  $\alpha,z$ 

- $\alpha, z$  ... is the azimuth and the zenith distance of the satellite,
- $\Delta \phi'(\alpha, z) \ldots$  models the elevation and azimuth dependent phase center variations,

 $\Delta \mathbf{r}$  ... defines the position of the "mean antenna phase center offset" with respect to the *antenna reference point*. This vector becomes uniquely defined by imposing the condition

$$\int_{\alpha=0}^{2\pi} \int_{z=0}^{z_{max}} \Delta \phi(\alpha, z) \sin z \, \mathrm{d} z \, \mathrm{d} \alpha = \min. \quad (\text{e.g., with } z_{max} = 75^{\circ}) \,.$$

The antenna reference points are defined for different antenna types in RINEX and IGS standards. For most antenna types the reference points are given in the file X:/GEN/ANTENNA.GRA.

e ... denotes the unit vector in the direction antenna $\rightarrow$ satellite.

The correction  $\Delta \phi(\alpha, z)$  is added to the geometrical distance  $\varrho_k^i$  of Eqn. (17.1).

In the *Bernese GPS Software*, two different model functions  $\Delta \phi'(\alpha, z)$  may be used:

- 1) Piece-wise linear function in elevation (and optionally in the azimuth): polygon approach.
- 2) Spherical harmonic function of maximum degree  $n_{max}$  and maximum order  $m_{max} \leq n_{max}$ :

$$\Delta \phi'(\alpha, z) = \sum_{n=1}^{n_{max}} \sum_{m=0}^{n} \tilde{P}_{nm}(\cos 2z) \left(a_{nm} \cos m\alpha + b_{nm} \sin m\alpha\right), \quad (17.3)$$

where  $\dot{P}_{nm}$  are normalized associated Legendre functions of degree *n* and order *m*, and  $a_{nm}, b_{nm}$  are the coefficients of the harmonic series development.

The receiver antenna phase center offsets  $\Delta \mathbf{r}$  and the coefficient defining the function  $\Delta \phi'(\alpha, z)$  are given in file X:/GEN/PHAS\_IGS.01. The first part of this file looks like:

MODEL NAME: IGS_01	, PHASE CENTER VAR	IATIONS	30-JUN_96
RECEIVER TYPE ANTENNA TYPE	ANTENNA S/N FREQ FROM TO L*	PHASE CENTER OFFSETS (M) NORTH EAST UP	FMT
*****	***** ****** *	** *** ** ** *** ** **	*
TRIMBLE 4000SSE	0 999999 1	0.0015 -0.0012 0.0751	2
TR GEOD L1/L2 GP	2	-0.0011 0.0017 0.0692	

In the first part of the file the phase center offset  $\Delta \mathbf{r}$  is given (north, east, and up components). Please note the difference between the offsets for the first and the second carrier. In the example file above, the antenna offsets are common for all the antennas of the same type (antenna numbers between 0 and 999999). The format flag (FMT) indicates that the antenna phase center variation model is given in the second part of the file:

REC	EIVEF	R TYPE		ANTENNA	TYPE		FROM	то	TYP	D(Z)	D(A)
***	****	******	*****	******	******	*****	*****	*****	***	***	***
TRI	MBLE	4000SSE		TR GEOD	L1/L2	GP	0	999999	1	5	360
	A∖Z	0	5	10	15	20	25	30	35	4	0
L1	0	0.00	1.80	4.60	8.10	11.70	14.50	16.10	16.90	16.9	0
L2	0	0.00	0.30	0.90	1.80	3.00	4.10	4.90	5.40	5.6	0

In this case, the model with piece-wise linear functions is used (TYP=1). The step in azimuth D(A) is  $360^{\circ}$ , which means that no azimuth dependence is considered. Let us point out that all phase center offsets and variations for different antenna types in file X:/GEN/PHAS\_IGS.01 are given *relative* to each other. This means, that one antenna type had to be selected as a reference by setting

$$\Delta \phi'(\alpha, z) = 0$$

It may be seen in file X:/GEN/PHAS\_IGS.01, that the antenna phase center offsets and antenna phase center variations are different for both carriers. The antenna phase center corrections for a linear combination of  $L_1$  and  $L_2$  observations are computed by forming the correspondent linear combination of  $\Delta \phi_{L1}(\alpha, z)$  and  $\Delta \phi_{L2}(\alpha, z)$ .

# 17.4 Estimation of the Receiver Antenna Phase Center Variations

The receiver antenna phase center offsets and variations may stem either from *chamber measurements* or from *estimations using GPS data*.

The chamber measurements are performed in anechoic test chambers, where one specific antenna is mounted on a positioner that may rotate the antenna around two independent axes and shift it in three directions. The transmitting antenna is kept fixed while the receiving antenna (to be tested) is rotated through zenith angles from -90 to +90 degrees for various azimuth values. To rotate the test antenna as precisely as possible around the "mean" phase center for the actual measurements, the antenna is first shifted with respect to the center of rotation, until the phase center variations with elevation are minimal and as symmetrical as possible for zenith angles corresponding to -z and +z degrees. Apart from recording the antenna phase values using a strip-chart recorder, the signal amplitude and axial ratio pattern are common measurements in chamber tests. These recordings have to be performed for both GPS carrier frequencies. Finally, the location of the center of rotation with respect to a physical point on the test antenna, e.g. the *antenna reference points (ARP)* as defined by the IGS, has to be determined. For more details on chamber measurements, we refer to [Schupler et al., 1994].

#### Estimation of Phase Center Variations Using GPS Data

The second possibility to determine the variations of the antenna phase centers is a GPS calibration campaign. The effort for setting up such a campaign is considerably smaller than collecting the chamber measurements. The resulting accuracy is comparable with that of the chamber tests, but the disadvantage is that only *relative* corrections (differences between antenna types) may be estimated. Therefore, at least one reference antenna has to be introduced as "known" (phase center offset and variations).

The *Bernese GPS Software* Version 4.2 provides the possibility to estimate the antenna phase center offsets, as well as the phase center variations. To activate the estimation, select an output file name in Panel 4.5.0 (ANTENNA PCV (GRID) for piece-wise linear functions and/or ANTENNA PCV (HARM) for harmonic coefficients) and enable the estimation in Panel 4.5–2.4 (RECEIVER ANTENNA OFFSETS resp. RECEIVER ANTENNA PATTERNS). If you estimate receiver antenna offsets you will be prompted with Panel 4.5–2.4.G,

4.	5-2.4.G	PARAME	ETER ESTI	EMATION:	F	RECEIVER	A	ANTENN	IA OFFSETS		
	Pacaivar Antanna Officat Daramators										
	Receive	er Antenna	UTISET 1	aramete	ers	3:			(		
	MODE	OF ANTENNA	A ESTIMAT	CION	>	GROUP		<	(INDIVIDUAL or GROUP)		
	FREQU	JENCIES			>	BOTH <			(L1, L2, or BOTH)		
	ESTIN	ATE NORTH	ANTENNA	OFFSET	>	YES <			(YES, NO)		
	ESTIM	ATE EAST	ANTENNA	OFFSET	>	YES <			(YES, NO)		
	ESTIN	ATE UP	ANTENNA	OFFSET	Ś	YES <			(YES NO)		
		INIL UI	ANTENNA	ULIDEI	1				(115, 110)		
	Dafaman	Deseine		. Dojm							
	referet	ICE RECEIVE	er/Anteni			NONE					
	REFER	RENCE RECEI	LVER NAME	5	>	NUNE			< (name, blank or NUNE)		
	REFEF	RENCE ANTEN	INA NAME	2	>				< (name or blank)		
	REFEF	RENCE ANTEN	INA NUME	BER	>		<		(number or blank)		
	Default	: A Priori	Sigmas:								
	HORTZ	CONTAL OFFS	SETS		>	0 0		<	METERS		
	VERTI		SETS		\$	0.0		ż	METERS		
	VEICE	ICAL OFFI	0110		1	0.0		`	FIET EILO		

where you may select the parameters and the reference receiver and antenna. You may decide to estimate the offsets for each individual antenna, or a common offset for all antennas of the same type (INDIVIDUAL or GROUP in Panel 4.5–2.4.H). The following Panel 4.5–2.4.I prompts you with the available antenna and receiver names available in the observation files, which you may modify.

4.5-2.4.I	PARAMETER ESTIMATION:	RECEIVER ANTENNA PATT	ERNS
RECEIVER NAME	ANTENNA NAME	ANTENNA NUMBERS FROM TO	SIGMAS (M) **.****
<pre>&gt; TRIMBLE 4000SSE &gt; ROGUE SNR-12 RM &gt; ROGUE SNR-8000 &gt; ROGUE SNR-8000 &gt; TRIMBLE 4000SSE</pre>	<ul> <li>&lt; &gt; 4000ST L1/L2 GEO</li> <li>&gt; DORNE MARGOLIN B</li> <li>&gt; DORNE MARGOLIN B</li> <li>&gt; DORNE MARGOLIN T</li> <li>&lt; &gt; 4000ST L1/L2 GEO</li> </ul>	<ul> <li>&gt; 23525 23525 </li> <li>&gt; 119 119 </li> <li>&gt; 20 20 </li> <li>&gt; 400 400 </li> <li>&gt; 67905 67905 </li> </ul>	> 0.00000 < > 0.00000 < > 0.00000 < > 0.00000 < > 0.00000 <

If you determine receiver antenna phase center variation patterns you are prompted with Panel 4.5–2.4.H allowing you to select the parameters and reference receiver and antenna names. In a panel similar to Panel 4.5–2.4.I you may modify the list of available receiver/antenna pairs.

4.5-2.4.H	PARAMETER	ESTIMATION:	RE	CEIV	YER A	NTE	ENNA	A PATTERNS
Receive MODE FREQU ESTIN	er Antenna Patt OF ANTENNA ESJ JENCIES MATION MODEL	ern Paramet TIMATION	ers > > >	GROU BOTH SPHE	IP I < CRIC	<	<	(INDIVIDUAL or GROUP) (L1, L2, or BOTH) (piece-wise LINEAR or SPHERICal harmonics)
# PO:	INTS / DEGREE 1	IN AZIMUTH	>	8	<			
Referen REFEI REFEI REFEI	nce Receiver/An RENCE RECEIVER RENCE ANTENNA RENCE ANTENNA	ntenna Pair: NAME NAME NUMBER	> N > >	ONE	<			< (name, blank or NONE) < (name or blank) (number or blank)
Defaul A PR	z A Priori Sign IORI SIGMA	nas:	>	0.0		<		METERS

If no reference antenna is specified, an a priori sigma for each parameter is required in order to prevent the normal equation system from becoming singular. If a reference antenna is specified, in general no a priori sigma is required, because the singular (0,0) term is not estimated in the spherical harmonics model and a special a priori constraint is put on the sum of all grid points for the piece-wise linear model.

The representation of the antenna phase center variations by a series of spherical harmonics is physically more meaningful, in particular for the azimuth-dependent variations, but the polygon model may be introduced more easily into other software packages (linear interpolation between tabular values). You may even estimate spherical harmonics coefficients and save the results in the form of a polygon model (see Panel 4.5.0 for output filenames).

It is also clear from Eqns. (17.2) that the station heights have to be fixed on known ground truth values. The remaining two components of the station position have to be fixed, too, if azimuth-dependent variations are determined. Fixing the horizontal position is not necessary if the antennas are rotated during the test campaign. The *Bernese GPS Software* handles this procedure by using the antenna orientation file. An example of such a file may be found in X:/INX/EXAMPLE.AZI:

RECEIVER ANTENNA ORIE	NTATIONS			01-JAN-94
RECEIVER TYPE ANTENNA TYPE ********	ANTENNA S/N FROM TO ****** *****	SESS ****	AZIMUTH DEG ***	
ROGUE SNR-8000 Dorne Margolin T	2104 2104	0011	270	
ROGUE SNR-8000 DORNE MARGOLIN T	3104 3104	0011	270	

The user may specify the name of the antenna orientation file in Panel 4.5. When estimating the phase center variations, one and the same set of coefficients may be estimated from several antennas of the same type (grouping of antennas).

We refer to [*Rothacher et al.*, 1995b] for details about the setup of calibration campaigns. The first problem which has to be solved is how to determine the correct station positions (ground truth). It

is, e.g., possible to use the following approach: let us assume that the calibration campaign consists of two sessions (the sessions should be quite long, at least 24 hours), and let us further assume that only two antennas were present. We select one antenna as reference. We want to estimate the antenna phase center variations

 $\Delta \phi'(z)$ 

(for the sake of simplicity, we do not assume any azimuthal dependence). Processing the first session (without estimating antenna phase center offsets and/or variations), the resulting relative position of both stations is corrupted by an unknown mean antenna phase center offset  $\Delta \mathbf{r}$ . If we exchange both antennas before processing the second session, the result of the second session is corrupted by  $-\Delta \mathbf{r}$ . The mean position (stemming from processing both sessions together) is correct. Now, it is possible to fix the station positions and to compute the coefficients defining the function  $\Delta \phi'(z)$ .

Let us include an example of PCV results thus derived [Rothacher et al., 1995b]:



Figure 17.1: Trimble antennas: spherical harmonics development of degree 10 estimated from GPS data.

17. Antenna Phase Center Offsets and Variations